1. The notion of infinity depends on the definition of finiteness. In modern mathematics (i.e. in Cantor's set theory), notions of finiteness and infinity refer to sets. Accordingly, infinite numbers (cardinal and ordinal numbers) characterize infinite sets.

Finite numbers measure finiteness. Finite numbers are positive integers. Considering their arithmetic, one can add these numbers, multiply and compare them in terms of lesser-greater. Accordingly, one can also add infinite numbers, multiply and compare them. However, the arithmetic of infinite numbers does not share all of the characteristics of finite numbers. For instance, addition and multiplication of ordinal numbers is not commutative, i.e. $a + b$ does not always equal $b + a$ and ab does not always equal ba. We find more differences when compare the arithmetic of infinite and rational numbers: there are neither negative, nor fractional infinite numbers, i.e., when a is an infinite number, then neither $-a$, nor $a/2$ are defined.

2. This project aims to develop a new theory of infinity. The mathematical aspects of the theory are based upon ideas introduced by L. Euler and J. Conway, while its historical perspective is rooted in Euclid's *Elements* and *Optics*.

The theory we propose includes Cantor's ordinal numbers, therefore it extends the current mathematical theory of infinity. In this theory, however, the definitions of operations differ from those provided by Cantor in such a way that, e.g., the multiplication of ordinal numbers is commutative. Moreover, in this theory one can consider both negative and fractional infinite numbers, and therefore, in this theory, there are negative and fractional counterparts of Cantor's ordinal numbers.

3. We will provide the historical, philosophical, and mathematical foundations of this theory. We will show that in ancient Greek mathematics, the notion of finiteness referred first of all to line segments. One could add those segments, take their fractional parts or compare them in terms of lesser-greater. Line segments also had the Archimedean property: when $a < b$, then by taking subsequent multiples of the segment a, such as $a + a$, $a + a + a$ etc., we obtain a segment greater than b, i.e., $a + ... + a > b$. Thus, at the very beginning of mathematics, finite objects shared the Archimedean property, and one could take their parts. This meaning of finiteness serves to be the starting point of our theory of infinity.

We will show that in the 17th century, the multiplication and division of segments was introduced. We will show, that in the 18th century, infinite segments (numbers), i.e., segments that violate the Archimedean property, were applied in mathematical proofs. One could add those infinite numbers, as well as multiply and compare them in terms of lesser-greater. Moreover, negative and fractional infinite numbers were also applied.

4. When a is an infinite number, then $1/a$ is an infinitesimal. Infinitesimals characterize non-Archimedean systems, which is why in the the title of our project, infinity stands in opposition to infinitesimals, rather than finite numbers, as it is in Cantor's theory. In our theory, infinite numbers from the very beginning belong to a structure known today as a non-Archimedean ordered field, as a result, the arithmetic of infinite numbers is the same as the arithmetic of fractions.