

Graphs and posets are ubiquitous in mathematics and theoretical computer science. They represent a way the given set of objects interact with each other. Usually, in a real-life (or scientific) application, we have some extra combinatorial, algebraic or geometric representation of a graph given on the input. For example, when we consider ranges of cellular network's antennas our objects are discs (covering the surface) and two antennas interfere (are in a relation) if they are within some short distance. Or given a family of axis-aligned rectangles on the plane we can consider a containment relation, i.e. one rectangle is smaller than the other if it is fully contained in it. This relation gives a partially ordered set defined in geometric means.

The most important measure of complexity of graphs is the chromatic number. It is the least number of colors we need to color the vertices (objects) of our graph such that no two adjacent vertices have the same color. Clearly, a graph with all the vertices pairwise adjacent to each other (a so-called clique) requires distinct colors on each vertex. This is nowadays a folklore result in combinatorics that there are graphs with no large clique but still with large chromatic number. Now, the punch-line of this project is that these graphs do not come up so easily in graphs defined in geometric means. For example intersection graphs of axis-aligned rectangles in the plane have chromatic number at most quadratic in the size of their largest clique. This was proven in a seminal paper by Asplund and Grunbaum in 1960 and it is an annoying long-standing open problem to improve this bound. On the other hand, Kraków's group a couple of years ago answered an old question by Paul Erdős whether intersection graphs of segments in the plane with no large cliques can have arbitrarily large chromatic number. The answer is positive and the construction witnessing it is called nowadays the Burling graphs.

Within this project we want to investigate further the influence of nice geometric representations of graphs (and posets) on the growth of the key measures of their complexity.

The way geometry interplays with various orders or orderings is a fascinating field of research in combinatorics. The studies of poset dimension, queue layouts, stack layouts, degeneracy, forbidden patterns are all part of it. As this is a broad field we decided, within this proposal, to pick three lines of research represented by some long-standing open problems.

For example, we want to pick up the challenge and try to improve the bounds from the above mentioned old paper by Asplund and Grunbaum. A progress here would be interesting even in some special cases when we restrict the given set of axis-aligned rectangles so that e.g. there are no two such that one is contained in the other.

This research project lies in mathematics and more specifically in combinatorics. We are struggling to prove or disprove conjectures that are known and published or that come up from various experiments we do on the way. We present our results to the community via conference presentations and journal articles.

The research topic is in the core of combinatorics and basically any progress in the listed long-standing open problems will be noted by the community.