

Description for the general public

The aim of the project is to study reflected backward stochastic differential equations (RBSDEs) with two optional barriers on general filtered probability spaces. The class of optional barriers is very wide and trajectories of such processes may be completely irregular. As for the filtration we assume that it is only right-continuous which covers standard models with dynamics given by Brownian motion and Poisson random measure.

The notion of RBSDEs is closely related to mathematical finance. In the models based on RBSDEs barriers play the role of pay off processes and filtration is related to the process describing the price of assets. The theory of RBSDEs also links to the so called obstacle problem for partial differential equations (PDEs). For example, when we have PDE with Laplace operator, then we have the so called heat equation which describes the variation of temperature in a given region over time. Then obstacles play the role of lower and upper barrier for temperature. Associated RBSDEs describe this model but on the level of particles. In the framework of RBSDEs the filtration is related to the Laplace operator and obstacles are related to the barrier processes.

Reflected BSDEs but with regular barriers and on the special class of filtrations, were subject of intensive investigation over the past two decades. The reason is that they have numerous applications in PDEs and mathematical finance. The notion of Reflected BSDEs was introduced in 1995-1997 by El Karoui with coworkers and Cvitanic and Karatzas.

Very recently mathematicians started the study of reflected backward stochastic differential equations with irregular barriers on general filtered spaces. The study of such equations is rather difficult, because irregularity of the data results in appearing new subtle phenomena and moreover most of the classical theory of processes concerns regular processes. On the other hand, by the same reasons, new very interesting and quite challenging mathematical problems arise.

In the project we would like to study RBSDEs with optional barriers, integrable data, general right-continuous filtration and nonincreasing continuous generator without growth condition. We would like to consider two cases: when barrier satisfies the so called Mokobodzki condition and the case without Mokobodzki's condition. The case without Mokobodzki's condition for irregular barriers will be considered in our project for the first time ever. This case leads to the entirely new notion of RBSDEs in which solutions may be irregular, however it is very important in application to the generalized Dynkin games.

In the study we would like to focus on some basic problems: existence and uniqueness of solutions, regularity and stability of solutions and relation of solutions with generalized Dynkin games.