Nonlinear integral forms related to jump-type Markov processes

The project falls within the intersection of stochastic processes, partial differential equations, and harmonic analysis: probabilistic methods will be used to solve the Dirichlet problem and to derive Sobolev regularity properties of the solutions. For diffusion processes, connected with differential-type Markov (local) operators, this is now a classical, textbook theory, but it is not so for jump-type Markov processes and their nonlocal generators. In recent years Kassmann, Ros-Oton, Vasquez, and collaborators introduced bilinear (Dirichlet) forms $\mathcal{E}^D$ of Lévy processes with the domain of integration restricted, to correctly state and solve the Dirichlet problem in spaces akin to L^2 (the set $D^c \times D^c$ is removed from the domain in integration). In a recent work of Bogdan, Grzywny, Pietruska-Pałuba, and Rutkowski we obtained sharp conditions for the solvability of the Dirichlet problem, in terms of the extension and trace theorems. We also proved the Douglas identity, linking the $\mathcal{E}^D-$energy of the solution with certain Sobolev-type energy of the external condition.

In this project, we plan to extend the results to general $p > 1$, possibly also to $p = 1$. To this goal, we will introduce and analyze nonlinear variants of Dirichlet forms of the process, $\mathcal{E}_p^D$, with the same restriction in the integrals domain as in the linear case. These forms will be denoted $\mathcal{E}_p$, $\mathcal{E}_p^D$.

Nonlinear integral forms, without domain restriction, have appeared occasionally in the literature in connection with L^p-Markov semigroups and their generators. Our research will add to a better understanding of the nonlinear forms related to jump-type Markov processes. We will obtain Douglas identities for the $\mathcal{E}_p^D-$energies. As a consequence of this identity we will prove extension and trace theorems for the corresponding Sobolev spaces, with $p > 2$ and geometrically regular sets D. We will use them to solve nonlocal, nonlinear Dirichlet problems similar to the porous medium equation.

We also plan to obtain optimal Hardy inequalities for the $\mathcal{E}_p^D$-energies, and to prove $\mathcal{E}_p^D-$variants of Hardy-Stein identities and Littlewood-Paley-Stein theory for Markovian semigroups and martingales.