

The proposed project concerns questions of the theory of *dynamical systems*. This is a relatively young branch of mathematics, whose origins go back to the beginning of the twentieth century and are related to the study of differential equations describing the movement of a number of bodies (for instance, the Sun, the Earth and planets) under the influence of gravity forces. It turned out that solutions of such equations cannot be described precisely by simple formulas and require different (*qualitative*) methods of analysis. Another element which caused development of this domain of research was related to questions of statistical physics concerning systems of many influencing bodies, like gas particles. To analyze such systems one needs to use stochastic (*ergodic*) methods arising from the theory of probability.

The theory of dynamical systems (in *autonomous* approach) studies long-term evolution of a given system, which can be described by time-independent deterministic rules. Mathematically speaking, this evolution can be defined by *iterations* (multiple compositions) of some transformation. In this approach, the time is measured by the number of iterates of this transformation and is called *discrete*. We are interested in what happens with typical points of the system after a long time, whether they behave in a *regular* (stable) or *chaotic* way, and what properties are shared by sets of points with similar behaviour. Allowing the case where the rule describing the evolution of the system changes in time, we define *non-autonomous* dynamical systems. A particular situation occurs, when the transformation describing this evolution is chosen every time in a random way. This leads to the theory of *random* dynamical systems.

The proposed project concerns the study of real and complex low-dimensional dynamical systems in both autonomous and random approach. This domain of research has experienced a rapid growth since the seventies of the twentieth century, becoming an attractive subject of studies for a number of mathematicians. The importance of this class of systems is also related to the fact that it is a source of many mathematical models describing phenomena which occur in natural and social sciences.

In the complex case we study holomorphic mappings of the complex plane. In the dynamical approach, this plane can be divided into two subsets – the *Fatou set*, where the iterations of the mapping behave in a regular way and the *Julia set*, where the dynamics has a chaotic character. It turns out that the Julia set usually has complicated topology and geometry. In particular, it is often *self-similar* in the sense that its small fragments are in some way similar to the entire set, and has other fractal properties. Within the project, we would like to study some elements of the so-called *thermodynamic formalism* (topological pressure and conformal measures) for *transcendental* mappings (of infinite degree), which enables a precise description of geometric properties of the Julia set and computing various kinds of its dimension. We will also study the question, how these dimensions change under the perturbation of the mapping. Among other considered tasks are the description of sets of points converging to infinity at a given rate and the study of topological and geometric structure of the Julia sets for some classes of transcendental mappings.

Another research task presented in the proposed project is the study of so-called *coarse expanding conformal mappings*. Examples of these mappings are given by branched coverings of the sphere of a special type, generalizing *Thurston maps*. We are going to develop the ergodic theory of such mappings studying, among others, the Markov partitions, symbolic dynamics, and measures related to the dynamics of the mapping. It is worthy to emphasize that this theory has deep relations with other branches of mathematics, like topology and geometric group theory.

Other tasks contained in the proposed project concern the dynamics of non-autonomous and random systems. We would like to examine in what way the introduced randomness influences the dynamics of real and complex low-dimensional dynamical systems. In the complex case, among the subjects of interest will be the dependence of topological and geometric properties (like dimension) of the Julia set on the introduced random perturbation. In the real case we will study the systems of several mappings of the interval and the circle, which are chosen randomly during iteration, with some time-independent probabilities. Under suitable assumptions such systems have the so-called *stationary measure*, which is useful in the description of their ergodic properties. Within our project, we are going to study the regularity (absolute continuity or singularity) of this measure for some classes of such systems.

The proposed research tasks are the subject of intensive studies of a number of mathematicians dealing with dynamical systems. The analysis of these problems and solving at least some of the open questions related to them would make a valuable contribution to the development of this branch of mathematics.