Description for the general public
Project: Continuous and discrete perspectives in harmonic analysis

The research proposed in the project lies in the mainstream of harmonic analysis. Harmonic analysis (or Fourier analysis) is a branch of mathematics that grew out of the theory of Fourier series. Its main idea is a decomposition of the studied object onto a sum of simpler components. Methods of harmonic analysis find broad applications not only in mathematics but also in everyday technology: in computed tomography, data compression (MP3 and JPEG formats), or in signal processing (for instance in radio waves).

In this project we will study discrete and continuous objects of harmonic analysis, with emphasis on their mutual interactions. Our questions fit in the fundamental question on the nature of the universe: is it discrete or continuous? The objects we are going to study in the discrete context are certain averages over convex sets (so called discrete Hardy-Littlewood averaging operators), which are defined on the integer numbers. We would like to understand what happens with these averages when the dimension (number of integer coordinates) grows. In this context our research has an interdisciplinary character, touching upon questions of analytic number theory in high dimension, such as the multi-dimensional Gauss circle problem, and multi-dimensional geometry of convex sets. We will also study similar questions on dimensional independence in the continuous settings. Here, the considered objects (so called Riesz transforms) have a close relation with operators appearing in partial differential equations. Finally, the last part of the project concerns so called multiplier operators. These operators are a mathematical model of filtering an electrical signal, allowing us to weaken or increase some of its aspects. In the project we will study spectral and spherical multipliers in some models of non Euclidean geometry.

Even a partial solution of the proposed research questions will allow us to better understand connections between harmonic analysis and multi-dimensional analytic number theory and geometry. In a broader context, the proposed investigations will lead to a better understanding of continuous and discrete objects of harmonic analysis and their interactions.