The goal of science is to understand and to describe as precisely as possible the world around us. To achieve this goal scientists construct theoretical models of phenomena under consideration, test them experimentally and theoretically, adjust them and estimate probabilities of future possible events. The primary language of science is the mathematical language, which is ruled by strict logic and inspiring intuition. It is based on numbers, vectors, tensors, functions, and a huge number of much more sophisticated abstract notions and tools. The mathematical language, like any other language, is a living substance, which does not remain unchanged. It constantly develops and is being enriched with new tools, ideas, techniques, and theories, and one of the latest powerful inventions in mathematics are *random matrix theory* and *random graph theory.*

A random matrix is a matrix whose entries are variable quantities taking different values depending on a set of random events. The study of random matrices has emerged from various applications. Firstly they were introduced in multivariate statistics by John Wishart for statistical analysis of large samples. Then John von Neumann and Herman Goldstine used them to describe computation errors. Independently, Eugene Wigner came to idea to use symmetric random matrices as statistical models for heavy nuclei. Nowadays random matrix theory has a huge number of various important applications in completely different fields such as nuclear physics, quantum chaology, condensed matter, statistical physics, quantum information theory, quantitative finances, and telecommunications just to list a few.

The explanation of such popularity of random matrices is simple because a lot of things in the real world are described in terms of operators or matrices (e.g. Hamiltonians). And whenever one sees a matrix which properties one does not know, one tries to think about a typical property and to write a random matrix possessing this typical property. The hope is that this model will predict other properties of the phenomenon one wants to describe. Such combination of typicality and a "black box" is a common probabilistic approach in general. Also, in random matrix theory one deals with multidimensional (matrix) integrals, while even one-dimensional integrals describe a lot of things.

The most important information about random matrices is contained in asymptotic and non-asymptotic properties of their *eigenvalues, singular values, eigenvectors, and singular vectors*, where by "asymptotic" we mean the regime when the dimensions of a sequence of matrices tend to infinity. For example, the eigenvalues of the model used by Wigner correspond to the energy levels of heavy nuclei. Another example is singular values of random matrices with independent entries. The largest and smallest singular values are especially important, they have been intensely studied, in part due to applications in theoretical computer science. Von Neumann and Goldstine, being motivated by their work on the first electronic computers, were looking for upper bounds for the so called *condition number*, which is a ration of the maximum to minimum singular values. The condition number measures how sensitive the solution of a system of linear equations is to perturbations in the input data and to roundoff errors made during the solution process. More recently, bounds on the condition number of non-centered random matrices found their applications in the theory of smoothed analysis of algorithms developed by Spielman and Teng. The smallest singular value has also received attention due to its connection with proving convergence of the empirical spectral distribution of matrices.

In the current project we deal with different types of random matrices: Hermitian and non-Hermitian, sparse and not sparse, having independent and slightly dependent entries. We are interested mainly in the quantitative estimates for the smallest singular values of these matrices, delocalization properties of eigenvectors and singular vectors, convergence of empirical spectral distributions, and in establishing the asymptotic normality of linear eigenvalue statistics. We expect that our research will have a substantial impact on advances in random matrix theory and its applications. The problems described in this proposal either take their origin or have applications in statistics, physics, quantum information theory, numerical analysis and random graphs theory. In particular, sharp estimates of the smallest singular value immediately yield better bounds for the condition number of the matrix which, in turn, is the crucial parameter in analyzing performance of various numerical algorithms. Overall, this project has the potential not only to develop fundamental knowledge on random matrices and random graphs, but also to benefit society through improving the theoretical basis for developing information technologies and computer software dealing with data processing.