Line arrangements and their geometric and algebraic properties.

Description for the public

Line arrangements are classical subject of study in geometry. They were studied in particular by Pappus of Alexandria who proved his famous theorem in the 4th century. At the end of 19th century Sylvester proposed to study arrangements of real lines with as small number of double points (points where only 2 lines intersect) as possible (preferably 0). To avoid trivialities he was interested in lines in projective planes, these are extensions of Euclidean planes where no parallel lines exist, i.e., each pair of lines has an intersection point. It was proved around 1940 by Gallai that for real lines there are always double points. Only recently Green and Tao proved that their number must not be less than half the number of lines (up to a short list of exceptional cases).

Böröczky constructed series of examples where the number of double points is equal or close to the lower bound given by Green and Tao. Such an example of 12 lines is depicted in Figure 1. Böröczky

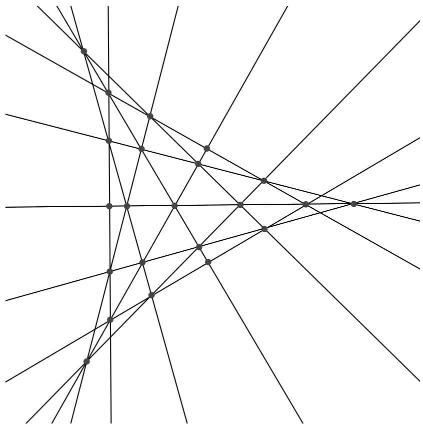

FIGURE 1. Böröczky arrangement of 12 lines.

arrangements attracted considerable attention recently when it was realized that their intersection points provide counterexamples to some important conjectures in commutative algebra. They have appeared also in the study of linear systems with imposed base loci and unexpected hyperplanes appearing in this study.

The purpose of this project is to study algebraic and geometric properties of Böröczky arrangements and some of their extensions, i.e., arrangements which contain them as proper subsets. The results should contribute to better understanding of containment relations between symbolic and ordinary powers of ideals and the relation between unexpected hypersurfaces and properties of sets of points giving rise to their existence.