From the first principles of quantum mechanics, microscopic systems are governed by the Schrödinger equation. This equation is *linear* but its complexity grows dramatically when the number of particles becomes large. Therefore, it is very important to rely on effective models, which have much fewer variables but usually are *nonlinear*. This gives raise to two major goals of mathematical physics: justifying the validity of effective models, and analysing the solutions of those limiting theories.

The topic of this research project concerns the macroscopic behaviour of interacting Bose gases. One of the most remarkable phenomena of Bose gases is Bose-Einstein condensation, which occurs when below a critical temperature most of the particles occupy a common quantum state. This phenomenon was theoretically predicted in 1924 by Bose and Einstein and it was first experimentally observed in 1995 in the Nobel Prize winning works of Wieman, Cornell and Ketterle. Since then, the rigorous derivation of the Bose-Einstein condensation from the first principles of quantum mechanics has become one of the fundamental questions in many-body quantum physics. In fact, Bose and Einstein only dealt with an ideal (i.e. non-interacting) gas and the presence of particle interactions complicates the analysis crucially. Heuristically, when the interactions are sufficiently weak, one can treat the particles as if they were independent but governed by a common self-consistent one-body potential. This approximation leads to the Hartree/Gross-Pitaevskii functional, which is expected to capture the leading order of the ground state energy and ground states of the many-body system. In this project we would like to understand the system beyond the leading order behaviour of the Bose–Einstein condensate. In 1947 Bogoliubov proposed an effective theory for the excitation spectrum and used it to explain the phenomenon of superfluidity. Roughly speaking, Bogoliubov's theory is a (second) quantized version of Taylor's expansion of the Hartree/Gross–Pitaevskii functional at its minimizer. Both the Hartree/Gross–Pitaevskii and Bogoliubov theories have their dynamical counterparts which describe the dynamics of a many-body boson system. The goal of the first part of the project is to derive and analyze the Hartree/Gross–Pitaevskii and Bogoliubov theories in the context of dynamics of many-boson systems.

The second group of problems concerns the behaviour of many-boson systems in the thermodynamic limit. In particular, we plan to investigate spontaneous symmetry breaking in such systems. Symmetry breaking is a crucial concept in modern theoretical physics. Loosely speaking, it describes a situation when a physical system has some symmetry in the initial state but loses that symmetry in the final state. Symmetry breaking can appear in many contexts, varying from condensed matter to high energy physics. It is very hard to capture rigorously. In this project we plan to make progress in that direction in the context of many-boson systems.