Quantitative study of set theory

Are ZF and ZFC asymptotically equivalent?

Probabilistic methods are crucial in solving hard computational problems in computer science. Instead of studying all objects, the probabilistic paradigm allows us to study *typical* ones. Although this may cause a loss of precision, in return we receive algorithms of better time complexity. The investigated problems can be formulated by means of the notion of *"random formulas"* or *"random terms"*. The natural formalization of this *"randomness"* is based on uniform probability distributions of sets of objects of bounded size. In this sense the definition of a random formula is consistent with the notion of the density, used, among others, in theory of formal languages. Given a fixed positive integer n, the frequency measure of occurrences of objects from a given set A is defined by the ratio of the number all elements from A that are of size n to the number of all considered elements of the same size. The limit of the sequence of such ratios as n tends to infinity (provided it exists) is called the density of set A.

Quantitative questions that can posed in this field are: How many proofs of size n does a propositional formula have? How many programs of size n have the halting property? It turns out that structures in which bindings of variables are involved are extremely interesting from the viewpoint of combinatorics. The simplest such structures are given by predicate formulas or lambda terms with the lambda abstraction operator as a binder. Such objects are also formulas of predicate logic, where there are two kinds of bindings of variables, general and existential quantifiers.

The question of the density of the provable formulas of this theory in the set of all formulas of the language in which they are expressed is striking. Below we list problems on which we plan to work within the project. The main goal of the project is:

(1) **Show the existence of density for the set theory ZF expressed in the predicate calculus with one binary symbol.**
(2) **Find the quantitative relationship between Zermel-Fraenk's ZF theory and Zermela-Fraenkel's ZFC theory with axiom of choice.** Assuming that there are densities of **ZFC** and **ZF**, we are going to show whether there is **no** asymptotic difference between the number of theorems in both theories. We believe that of **ZFC** and **ZF** are asymptotically identical. Before that we have to answer the following basic questions:
(3) **Enumerate formulas of predicate logic, research on asymptotic density.**
(4) **How many closed formulas asymptotically we may build for the given size?**
(5) **What is the density of a set of formulas in the calculus with one binary predicate.**