SENSITIVITY ANALYSIS OF NONLOCAL OPERATORS WITH APPLICATIONS TO JUMP PROCESSES

DESCRIPTION FOR THE GENERAL PUBLIC

Objectives: many physical and social phenomena can be modelled as transportation of particles in space. The transportation may be continuous, similar to the erratic movement of pollen in still or moving air or water (the Brownian motion) or discontinuous, with jumps gradually arriving in time. The latter occurs, e.g., when trapping or tunelling of the particles take place. A typical example are changes of a system with a finite number of states, which cannot be continuous. Another one is the sloshing of the surface of water, where forces are tunelled by deeper layers of the liquid. It can be argued that many real-life phenomena, like weather patterns or prices of stocks exhibit jump-type behavior. In fact, from a certain mathematical perspective the jump-type phenomena–understood as Lévy-type stochastic processes–may be considered more general than the continuous phenomena–understood as diffusion processes.

As proper for mathematical modelling, in this project we attempt to capture fundamental features of jump dynamics and resolve critical difficulties by proposing an abstract but flexible mathematical framework. This is intended to offer a unifying perspective for complicated phenomena. In such abstract setting it turns out that specifying the intensity of jumps does not necessarily (uniquely) define the target dynamic. We need to put considerable effort to actually construct the mechanism and describe its qualitative and quantitative properties before they can be made useful in theory and in modelling. An interesting feature of our proposal is that we plan to use advanced methods from Partial Differential Equations to the theory of Lévy-type processes. To this end we will combine the expertise of the work group of René Schilling from Technische Universität Dresden and of the work group of Krzysztof Bogdan from Wrocław University of Science and Technology.

Research to be carried out: From specified intensity of jumps we will construct the dynamics of the Lévy-type processes in positive time. We will describe how the dynamics depends on the starting point and how it depends on the intensity of jumps, which may fluctuate in space and time. The results will be used for approximation of jump processes and statistical analysis of stochastic processes based on a finite number of observations. We will also examine the behavior of the process in large time, to describe global tendencies and predictions.

Reason for choosing the research topic: This research project belongs to the theory of stochastic processes. However, because of numerous and deep links with other areas of mathematics the outcome of the project will also be significant for potential theory, the theory of non-local partial differential equations, as well as statistics and financial mathematics. Selected results may also find practical applications, since the considered processes underly many models used in mathematical physics, statistics and finance for discontinuous phenomena observed in continuous time.