

Model theory is a branch of mathematical logic focusing on the study of mathematical structures which are *models* of a given theory, with the word *theory* having a precise meaning established by mathematical logic. More specifically, model theory is concerned with comparison, classification and studying the structure of such models. The key notion of the studies is that of definable sets. A *definable subset* of a model (or its Cartesian power) is a set that can be defined using the language of the ambient theory; more formally, it is the set of elements of a model satisfying a given formula of the language. A fine example is the solution set of a given system of equations, e.g. polynomial or differential equations, depending on the language being considered.

The project aims to explore various questions regarding groups in model theory. In particular, it will entail verifying the existence of a model companion of a theory of fields with certain group actions, and the study of model-theoretic properties of this companion. The existence of such model companion is closely linked with the existence of “ideal” (or “universal”) models of this theory, i.e. models that contain the solutions of all “potentially solvable equation systems” as well as infinite systems of such “equations”. Good properties of the model companion can yield applications in other areas of mathematics.

The project also aims to study various properties of definable groups and automorphism groups of models of a given theory using the methods of topological dynamics. The planned studies will involve methods of algebra, topological dynamics, combinatorics and model theory. The goal here is to settle a number of fundamental conjectures regarding spaces of strong types (i.e. spaces of classes of certain invariant equivalence relations in a sufficiently large model of a given theory) and a certain canonical group arising from topological dynamics (the so-called Ellis group). Those conjectures appear mostly in the context of so-called simple theories and theories with the independence property. These classes of theories have been intensively studied in the past twenty years and provided many important examples and applications, i.e. to diophantine geometry and combinatorics. Another aim of the project is to use the methods of topological dynamics to extend some key elements of the stability theory (i.e. a very well-developed theory of models in the class of so-called stable theories) to much larger classes of theories.

With a definable group one can associate its canonical quotients, which are compact topological groups. Those quotients play an important role in model theory and some of its applications. Therefore, another aim of this project is to define and study other quotients of this kind and to find an explicit description of those new (and those previously defined) quotients for some classical matrix groups. This, in particular, will lead to explicit descriptions of some classical compactifications of the considered groups.