The project is in a central subject of algebraic geometry. Algebraic geometry is the study of algebraic varieties which are sets of solutions of systems of polynomial equations. Such context is special enough to acquire tools from different parts of mathematics such as algebra, analysis, number theory, differential equations and rich enough to model problems not only in other branches of mathematics but also in theoretical physics. For this reason algebraic geometry is among the most studied branch of mathematics. The leading problem of algebraic geometry is the problem of classification of all algebraic varieties. However, algebraic varieties form a so vast collection of objects there is no hope to classify them in general. Instead, one restricts to special classes of varieties which are important for various reasons. Among the most important types of algebraic varieties one find varieties with trivial canonical class, so called Calabi–Yau type varieties. Their importance in algebraic geometry follows from the special place such varieties occupy in the classification theory; they are in some sense on the boundary between the better understood class of Fano type varieties and the varieties of general type for which there is no hope of general understanding. The classification of Calabi–Yau type manifolds starting from dimension 3 is one of the biggest challenges in algebraic geometry. Let us point out that all Calabi–Yau type varieties can be built using three classes of varieties: Calabi–Yau manifolds, tori, Hyperkähler manifolds. The motivations for the study of varieties with trivial canonical class go far beyond the problem of classification. As very natural objects these varieties appear in different contexts in many branches of mathematics and theoretical physics. Our research focuses on the study of Calabi–Yau manifolds and Hyperkähler manifolds.

One of the main motors of recent development of the theory of Calabi–Yau manifolds is the fact that they are used in physical string theory to model the shape of the universe. Roughly speaking string theory postulates that the universe is fibered by tiny Calabi–Yau threefolds in which there are vibrating strings. The behavior of the string on the Calabi-Yau threefolds determines the type of particles that we observe. For that reason understanding Calabi–Yau varieties is crucial for the understanding of the string theoretical model of the universe. The theory of Calabi–Yau threefolds is developed in parallel with that of string theory and as such it is full of conjectures motivated by physics. The most famous is the mirror symmetry conjecture whose mathematical counterpart postulates that Calabi–Yau threefolds arise in pairs having some of their structure interchanged. Some mathematical versions of mirror symmetry have been proven for the easiest to handle types of Calabi–Yau threefolds, so-called complete intersections in toric varieties. Outside this class the conjecture is widely open and motivates the need to find new non-standard constructions of Calabi–Yau manifolds. In the world of Calabi–Yau manifolds there are also other intriguing symmetries that are not yet understood. For example wall crossing in so-called GLSM theories may give rise to pairs of Calabi–Yau manifolds which are in some sense equivalent. Conjecturally, such manifolds should be so-called Fourier–Mukai partners and should admit the same mirror. Furthermore, it appears that these partners might also be related by topological cut and paste construction via so-called $\mathbb{L}$-equivalence. Since the theory of GLSM and wall crossing in so-called abelian GLSM gave crucial insight into the proof of mirror symmetry for toric complete intersections, we expect that the understanding of this phenomenon should give new insight to the mirror symmetry in general. Our aim is to provide new natural constructions of Calabi–Yau manifolds related to GLSM with non-abelian gauge group and investigate wall crossings in order to prove Fourier-Mukai duality and $\mathbb{L}$-equivalence of different phases partially solving conjectures in the subject.

Hyperkähler manifolds are mysterious manifolds that are very hard to construct, so hard that there appeared in the literature a (false) proof that such manifolds do not exist in dimension higher than 2. Nowadays, we know only 4 types of hyperkähler manifolds, but even in these types the construction of non-trivial projective elements is a challenge. There are up to now six known constructions of complete families of projective hyperkähler manifolds and all of them are related to deep and beautiful geometry. Let us point out that all these known families are in in fact of the same type called $K3^{[n]}$. Our project aims at finding new types of constructions but also providing some general setup for the construction of such varieties. In particular we plan to find the first example of a complete family of hyperkähler manifolds of generalized Kummer type (these are not of $K3^{[n]}$ type). Hyperkähler manifolds are also very natural objects and as such are related to other problems and the relation is sometimes very surprising. For that reason we investigate the geometry ans other features of known and new classes of hyperkähler manifolds.