Anomalous stochastic dynamics in the underdamped regime

Popular science summary

The interaction of a particle with the surrounding environment is often too complicated to be described exactly. In this case, it is much more convenient to approximate them using noise, i.e. interaction, which at any moment takes a random value. The motion of a particle under the influence of random force is called stochastic dynamics, and the branch of science describing it – a theory of stochastic processes. Trajectories of such a process are very irregular. They are (surely) continuous but non-differentiable at any point. The standard application of the theory of stochastic processes is the description of the Brownian motion. However, this is not the only application of random processes. Movement of people, dispersal of banknotes, searching (foraging) strategies of animals, chemical reactions, light scattering, diffusion in turbulent media and many processes in body cells can be described or approximated by random processes.

Popular meaning and use of the term "noise" suggests that it is undesirable and one should reduce or completely eliminate it. However, surprisingly it is not the case. The noise can amplify the system's response. There is a number of phenomena that demonstrate the constructive role of noise and fluctuations. Moreover, these phenomena would not be observable and detectable in the absence of noise. Finally, medical (therapeutic) applications of noise are also documented, for example, in the post-stroke rehabilitation.

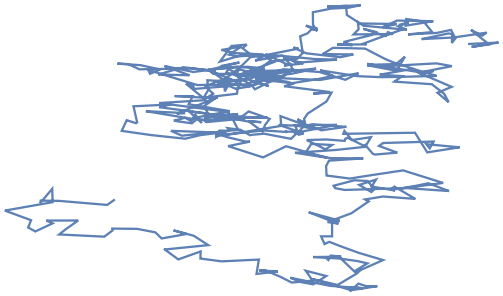

Figure : Sample trajectory of Brownian motion.

The random nature of stochastic dynamics causes that we can no longer accurately determine the position and velocity of the particle. We can only consider the probability that the particle is located in a specific place and move at a certain velocity.

The environment affects the particle not only by noise, but also by the frictional force (damping), i.e. "unwillingness" of the particle to move. Describing a particle in a stochastic process, it is often assumed that this force is so large that the movement takes place under the influence of "pushes" generated by noise only – the movement is overdamped. We know much less about the behavior of particles under the influence of noise in the underdamped regime, when the aforementioned assumption is not met.

The main aim of this project is to investigate noise-induced phenomena in the regime of the full (underdamped) stochastic dynamics. In this regime, stochastic systems are characterized by the position and velocity. Therefore, contrary to majority of earlier research focused on the overdamped dynamics, these studies are not limited to examining the position and its distribution. Planned investigations will examine full stochastic dynamics. This project will explore processes of noise-induced escape from finite domains i.e. classical phenomena demonstrating the constructive role of noises in physical systems. The proposed extensions assume more general noise than so far used and additionally account for nonlinear friction. Such studies will allow better understanding of the microscopic dynamics of single particles and molecules, which are inherently associated with noise since these objects are constantly interacting with other particles and molecules in their environment. Finally, performed studies will contribute to better understanding of the dynamics of biological systems in the microscale.