Collective behavior – a mathematician's perspective

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Popular descripion

To understand properties of dynamics of collective behavior let us think about a group of individuals like animals. Consider the following situation, we want to examine the case the whole group finally follow one direction. This idea has been simple illustrated for flocks of birds. The goal is to model the process of creation a regular flock like V-shapes. Roughly speaking, from a very chaotic configuration of individuals in the phase-space, not only in the position, but also in their speeds, there creates a regular flock with some well-ordered structure.

From the viewpoint of mathematics we study the following phenomena. Individuals see each other and they want to align their speeds, they want to form a group, be close together, but they want to avoid collisions. The goal of mathematics here is to check if a proposed model indeed fulfilled the expected properties characteristic for the studied phenomenon. That is the reason the qualitative analysis is the most desired in the investigation of such systems. For instance, keeping in mind example of birds, we shall check if V-shapes are indeed stable.

At the level of the choice of the model we meet the issue of scale, the model should be related to the number of individuals, and the time scale in which we plan to describe the phenomenon. Based on the classical Newtonian approach we consider three kinds of descriptions:

– As the studied group is small, the system can be prescribed by a set of ordinary differential equations (sometimes with memory or different non-local properties), here the character and main features of dynamics should be seen the easiest.

– Next, as the group is bigger, we have the kinetic description, where we look for a distribution of individuals in the space and filed of velocities (Vlasov type equations).

– Finally, we have the hydrodynamic models, connected with large "dense" group of individuals.

To illustrate the above cases let us take the example with birds. The first kind is related to description of a small group of birds, the second one can be appropriate to the case of large group of birds which starts to fly, and the last one should model the case of large "dense" flock, where it is impossible to distinguish a particular individual.

What we investigate the above systems for? From the viewpoint of a mathematician, the above models are interesting since they deliver novel qualitative results, which are not possible to gain for the classical physical systems, they require to construct/discover new analytical theories. Moreover, applications of such system one can find in economics, sociology or even in forensics and robotics.