Description for the general public

Groups are some of the most elementary and important objects in mathematics and appear practically in every mathematical problem. Consequently, they are useful in other branches of science: biology, chemistry, physics, and recently also increasingly in computer science.

The current project is concerned with the study of groups — which are a priori algebraic objects — using the methods coming from geometry and measure theory. This is quite a new field, which has been developed within the last thirty years, drawing on the problems and methods from differential geometry, algebraic topology and combinatorial group theory, among others. Roughly, one can say that we are trying to endow a group with a metric or measure structure in order to deduce some algebraic properties of the group using the latter structure. This situation is a beautiful example of when two seemingly unrelated approaches to a problem — geometric and algebraic — used together give very strong tools. Indeed, looking at groups as geometric objects has lead to breakthrough discoveries concerning their structure, for example in the case of outer automorphisms of free groups.

In this project we concentrate on several important classes of groups. *Coxter groups* are a generalization of groups of reflection symmetries in the Euclidean space. They are a very important source of examples both in group theory and, for example, in differential geometry. *Artin groups* are groups, which include for example the braid groups, as well as some mapping class groups of surfaces. Artin groups have been intensely studied for a long time and, hypothetically, they have a series of interesting properties but they hold their secrets very well guarded. *Polynomial automorphisms groups* of the affine space are important objects in algebraic geometry. A great progress in their study has been achieved recently using the methods of geometric group theory. Groups which are *hyperbolic in the sense of Gromov* form a huge class of groups containing many classical objects, which were previously studied separately. It contains, for example, both the non-abelian free groups and the fundamental groups of closed negatively curved manifolds.

The above mentioned classes of groups will be studied from several different angles. The Coxeter groups will be endowed with a structure, which will allow us showing the *biautomaticity*. This is a very strong algorithmic property, which has many algebraic consequences and, hypothetically, is true for Coxeter groups. This has been proved, however, only in few cases. We will also try to show that the *Conjugacy Problem* is solvable in the class of Coxeter groups. On the other hand, we will construct new examples of Coxeter groups with exotic properties. For other classes of groups, we will try to prove the so-called *Tits Alternative*. This is a property hypothetically possessed by all so-called non-positively curved groups. This was established, however, only in very few special cases.

Finally, we are going to use measure-theoretic properties of groups and study their measurable actions on probability spaces. This part will be connected with logic and an old problem of Tarski concerning the *squaring of the circle*. We are going to use the methods of measurable group theory to give general necessary and sufficient conditions for when two measurable subsets (e.g. disc and square) of the same measure are equidecomposable with respect to an action of a discrete group acting on the space where the subsets are located.