Popular summary of the project: Mathematical Fluid Mechnics

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The objective of the project is the mathematical analysis of the models of fluid mechanics (for liquids and gases). This field is an important branch of applied mathematics, meaning that theoretical results have a natural interpretation in different domains of science, in physics, biology, chemistry or social sciences. In order to understand the importance of our research let us focus on a classical example, the most famous open problem in the theory of partial differential equations - *The VI Millennium Problem.* The question is whether so called weak (of low regularity, possibly non-unique) solutions of the Navier-Stokes (NS) equations become classical (regular, unique) for regular initial data. In spite of simple formulation the problem remains unsolved for nearly 90 years. As mathematicians we expect that its eventual solution would give rise to completely new theories of PDEs and establish new connection with different domains of science. From the point of view of physics, it would give an answer to the fundamental question if NS equations can model turbulent flows. It would finally facilitate considerably numerical implementation of algorithms based on NS. At the same time, one of the most advanced results for the classical Euler equations (E) (C. DeLellis, L. Székelyhidi, 2009) shows existence of very weak, highly nonunique solutions with nonphysical properties (for example the energy of the system may appear and disappear). Two above examples show how mathematicians can contribute to deeper understanding of the models, developing at the same time highest level mathematics.

Our research will be focused on systems based on Euler and Navier-Stokes equations describing simple fluids as well as structured systems which allow to incorporate more complex flows. In order to understand better the mathematical essence of the problem it is often useful to investigate reduced systems or partial problems. They contain sufficient information to determine the admissibility of the original system, reveal interesting qualitative properties of solutions or can give efficient algorithms solving the generic system (for example in equations related to image processing), being at the same time simple enough to admit rigorous mathematical analysis. As applied mathematicians we aim at determining how and when a simplified system can replace more complicated original model, and identifying the keynote features of the system determining physically observed properties.

From mathematical point of view the systems which we investigate are not of a well defined type, therefore general well-developed theories cannot be applied directly. We are forced to combine them and develop new mathematical tools which will be applicable in other disciplines of science. The issues to be investigated are regularity of solutions, their uniqueness and descriptions of qualitative properties of given system. All these features are indispensable for physical interpretation. The problems we plan to tackle are among interests of leading research groups. Spam solutions (DeLellis-Székelyhidi) appeared at the end of the previous decade, new techniques for weak solutions to compressible models (Bresch-Jabin) in last years. Our goals however are not limited to cooperation with leading researchers, we aim at creating the mainstream of research, and recent achievements of Danchin and Mucha proves our ability to do this. Realization of the project shall bring breakthrough results which will give rise to new trends in nowadays applied mathematics.