WEIGHTED INEQUALITIES FOR MAXIMAL OPERATORS

The Hardy-Littlewood maximal operator M applied to a function f picks, at every point x, the greatest of the averages of the function f among a certain family of sets containing x. This way we obtain a new function Mf, which bounds f from above.

Maximal operators are useful tools in the study of various inequalities in harmonic analysis, which have further far-reaching applications in other areas of mathematics. For example, maximal functions majorize pointwise a large class of convolution operators, which can be exploited in the study of Hilbert transforms.

The objective of the present project is the study of estimates of the size of Mf depending on the size of f. The idea of size of a function can be formalised mathematically in various ways. The most popular one are norms in L^p spaces. There are numerous known results concerning estimates in L^p. In our project we want to generalise some of those results to weighted Lorentz spaces.

In our work we will apply the Bellman special function method. It rests on coming up with an appropriate function, enjoying certain concavity and majorization properties, and deducing the given estimate from those properties. This approach is not easy, especially in the case of Lorentz spaces, where it can be applied only indirectly. The fact that we deal with weighted spaces complicates the analysis even further, as the presence of weight introduces two extra parameters in the Bellman function we look for.