The scope of the project is modelling dependence between the components of multivariate Markov processes, in which the time parameter was changed in a random way. By a Markov process we mean a process whose future behaviour depends on the past only through the present. I means that future evolution of the process depends only on the current state, not on the path the process followed before. An example of a Markov process is a Brownian motion, that is a movement of a particle on water surface. Markov processes are commonly used for modelling phenomena in physics, biology, chemistry or economy.

When $X = (X_1, \ldots, X_d)$ is a random vector in $\mathbb{R}^d$, the problem of studying dependence between its components boils down to finding an appropriate function (so called copula function). More precisely, the Sklar theorem states that there exists such a function C, that the cumulative distribution function of the vector $X = (X_1, \ldots, X_d)$ is equal to $C(F_1(x_1), ..., F_d(x_d))$, where $F_1, \ldots, F_d$ are the marginal distributions of the coordinates. For example, if $X_1, \ldots, X_d$ are independent, $C(u_1, \ldots, u_d) = u_1 \cdots u_d$. Using this method we can separate the dependence structure of the vector from its marginal distributions. One may ask whether we can use the same reasoning when instead of a random vector, we consider a d-dimensional stochastic process. It appears that in this case we cannot describe dependence structure through a function of marginal distributions. In order to describe the dependence structure of a multivariate processes we need to find some relations between the characteristics of its components. If X is a d-dimensional Markov process an interesting question is whether its components are themselves Markov. Obviously it is true when the coordinates are independent; however, in general case it may not hold. A Markov process whose components also have Markov property are called Markov consistent.

The aim of the project is to study the influence of the random change of time on the dependence structure of a multivariate Markov process. By a random change of time we mean that the process is evaluated not at time t, but at time $\tau(t)$, where τ is a nonnegative, nondecreasing stochastic process, possibly dependent on X. We obtain a new process $Y(t) = X(\tau(t))$. For example we can imagine that when the process X jumps to the fixed state, time accelerates - so the new process will spend there less time, or other way around - in another state times slows down, making the process Y spend there more time (to illustrate it, imagine that time slows down every time we are on vacations and accelerates when we are at work). Such changes of time are often used to represent complicated processes as time-changed simpler processes, moving the complicated structure to τ itself. The question arises whether the process τ can contain information of the dependence structure of Y. In other words, given X and τ, what can we say about the dependence of the coordinates of Y - for example, whether the new process is Markov consistent. We can also reverse this question and ask whether there exists such a change of time τ that the process Y (in particular, its dependence structure), will satisfy certain conditions (for example, if X is not Markov consistent, can we find such a change of time that will impose Markov consistency on Y). In order to answer these questions we will study the characteristics of the process Y after various types of change of time.