Hardy's inequality

Various equalities and identities play a crucial role in mathematics. More precisely, it is interesting for the mathematicians when one quantity depending on a parameter is equal to other quantity, which also depends on that parameter but in seemingly different way. The generally known example of such equality is *Pythagorean theorem*, which says that in any right-angled triangle the square of the length of the hypotenuse is equal to the sum of squares of the lengths of the other sides. In this case the changing parameter is the right-angled triangle. Other very popular example is *Pythagorean identity*. It states that the sum of squares of sine and cosine of an angle equals one. Obviously, both the functions do depend on the angle, whereas the sum of their squares does not.

Besides equalities the inequalities are very important for mathematicians. They essentially appear when among two changing quantities one is greater (way call such inequality strict) or not less (then the inequality is weak) than the other. As a popular example may serve a property of the sides in a triangle. The sum of the length of any two sides of a triangle is (strictly) greater than the length of the third one. This quality is known as the *triangle inequality*. It is worthwhile to employ one more example, this time excerpted from the physics. In 1927 Werner Heisenberg observed and stated a rule, which is referred nowadays as the *uncertainty principle* or the *Heisenberg inequality*. It describes a phenomenon of the nature of the reality. Namely, it is not possible to determine the position and the momentum of a particle simultaneously. The product of the errors is always (weakly) grater than the Planck's constant divided by 4π. To sum up, inequalities play equally important role as identities, and moreover, as it turns out, appear in the nature and in the mathematics more often.

The main objective of this project is to study an inequality known in the world of mathematics, or more precisely in the world of harmonic analysis, as *Hardy's inequality*. It descends from the known British twentieth-century mathematician and the cricket fan Godfrey Harold Hardy. The compared quantities are the sum of expansion coefficients of a (generalized) function in a given orthogonal basis and a magnitude of the functions measured by certain norm. The mentioned basis may be a system composed of infinitely many functions, which has the following property: knowing only the expansion coefficients in the said basis of a function from a large function spaces it is possible to reconstruct the original function. Various orthogonal settings appear very often in mathematics and such reproducing properties are essential in many argumentations. The priority of this project is to establish the form of Hardy's inequality in multiple contexts, which occur in other fields of mathematics and physics such as the differential equations. The applicant's intention is to obtain a result possibly general and abstract, which might be useful in some estimate that are often necessary to solve some different problems. The foremost aim is the full characterization of Hardy's inequality on the *spaces of homogeneous type*, which have been fundamental in mathematical analysis for the last five decades.

The key reason to raise the described issues is the wish to obtain an inequality, which might be used in various problems emerging from the harmonic analysis. Such task requires a development of the known mathematical techniques and creation of the original methods. The better understanding of the relatively new concepts in the world of science, such as the above-mentioned spaces of homogeneous type, is an additional benefit, which may result from the realization of this project and consequently enrich the specialistic literature. Moreover, an indubitable advantage of answering the posed questions is the close relation of the considered object with physics.