DESCRIPTION FOR THE GENERAL PUBLIC

1. Objective

The formalization of mathematics and logic has had a huge impact on society in the 20th century: for example, the computer revolution, the rise of internet and mobile technology. The earliest formulation of the corresponding problem for scientific theories was Hilbert's 6th Problem:

> 6. *Mathematical Treatment of the Axioms of Physics*. The investigations on the foundations of geometry suggest the problem: To treat in the same manner, by means of axioms, those physical sciences in which already today mathematics plays an important part; in the first rank are the theory of probabilities and mechanics. (Hilbert 1900 [2])

However, this is a difficult, unresolved topic. Important works include those by Bertrand Russell, The Vienna Circle, Rudolf Carnap, Karl Popper and Alfred Tarski. The subsequent literature is large and often takes very different approaches. The project's objective is to investigate the formalization of, and the nature of mathematical representation in, scientific theories, using methods based on the mathematical notion of a **model** ([4]): *a system of objects, with various operations on them, and relations amongst them*. In logic, models arise as interpretations of formal axiom systems. In science, models arise typically as *representations* of physical systems. Much of my work since my PhD (Ketland 1998 [3]) falls into this area. A monograph has appeared on this topic: *Philosophy and Model Theory* (2018), by Button & Walsh ([1]), a good example of the kind of work the project intends to develop.

2. Research to be conducted

There are three main areas which the project will examine:

(a) Identity & Indiscernibility in Logic, Mathematics & Science
(b) Formalization of Scientific & Mathematicized Theories
(c) Representation & Structure

For (a), there are both technical and conceptual questions that arise in connection with the idea of objects a, b in a model M being "indiscernible". The last 10 years has seen significant advances in this field, some by the author and much of it summarized in Button & Walsh 2018. Several open questions remain in logic and in the foundations of mathematics & physics. For (b), progress has been made in understanding two major aspects of scientific theory formulation: the notion of *empirical adequacy* of a theory (and its relation to the Ramsey sentence formalization of a theory) and the role of *mathematicization* (e.g., representation/uniqueness theorems, which analyse how physical objects are mapped to abstract values). Further investigation of both matters is part of the project. For (c), the concepts of *abstract structure* and *structural representation of a physical system* have played a significant role in recent work in foundations of mathematics and physics. The project aims to give precise analyses of both notions, and apply the results to problems in these foundational areas (e.g., the physical equivalence of mathematical representations; gauge equivalence).

3. Reasons for choosing the research topic The listed areas lie at the cutting edge of recent state of the art research in the conceptual foundations of mathematics and physics. For example, the notion of *identity types* in type theories and category theory; the notion of indiscernibility in the *quantum physics of identical particles*; the nature of *structural representation in gauge theories*.

References

[1] Button, T. & Walsh, S. 2018: *Model Theory and Philosophy*. OUP.
[2] Hilbert, D. 1900. *Problems presented at the Paris conference of the International Congress of Mathematicians* Sorbonne 1900. Published as "Mathematical Problems", *Bull. Am. Math. Soc.* (1902).
[3] Ketland, J. 1998: *The Mathematicization of Nature*. (LSE, PhD Thesis)
[4] Marker, D. 2002: *Model Theory: An Introduction*. New York: Springer.