Let us fix a prime number p and a nonzero rational number x. Then x can be uniquely written in the form $\frac{a}{b}p^t$, where $a \in \mathbb{Z}$, $b \in \mathbb{N}_+$, $p \nmid ab$ and $t \in \mathbb{Z}$. The exponent t in the above representation is called the p-adic valuation of the number x and we denote it by $\nu_p(x)$. In case of $x = 0$ we put $\nu_p(0) = +\infty$. Then we define for any rational number x its p-adic norm $|x|_p := p^{-\nu_p(x)}$. Let us note that the p-adic norm gives a metric space structure on the field $\mathbb{Q}$. The completion of $\mathbb{Q}$ with respect to p-adic norm is a topological field and it is called the field of p-adic numbers $\mathbb{Q}_p$.

Study of p-adic valuations of terms of a fixed sequence of rational numbers is in general a nontrivial and interesting issue. Many mathematicians are working on it. The knowledge of the p-adic valuations of terms of a fixed sequence allows us to state if they are integer and helps us with solving some Diophantine equations. If $n \in \mathbb{N}$ then $H_p(n)$ is the number of solutions of the equation $\sigma^p = id$ in the group S_n. A qualitative description of values $\nu_p(H_p(n))$ was obtained for any prime number p. Let us notice that $H_p(n)$ is exactly the number of permutations $\sigma \in S_n$ which can be written as a product of pairwise disjoint cycles of the length p. In the same way one can define for any nonnegative integer $d \geq 2$ the number $H_d(n)$ as the number of permutations $\sigma \in S_n$ which can be written as a product of pairwise disjoint cycles of the length d. It is worth to note that there is not known a description of p-adic valuations of the numbers $H_d(n)$ if $p > d$ (even for $d = 2$).

For a set $A \subset \mathbb{N}$ we define its ratio set as $R(A) = \left\{\frac{a}{b} : a, b \in A, b \neq 0\right\}$. The results on denseness of sets $R(A)$, where $A \subset \mathbb{N}$, in the set of positive real numbers are known for several decades. On the other hand the issue of ratio sets of subsets of $\mathbb{N}$ in the fields of p-adic numbers is relatively new. Hence it contains many unsolved problems, for example p-adic denseness of the sets $R(A)$, where A is the set of values of a given quadratic form or the set of sums of n powers of non-negative integers with exponent 4 or 5, where $n \in \mathbb{N}_+$ is fixed. Another open problem in this area is the question about existence of a partition of $\mathbb{N}$ into two subsets A and B such that for each prime number p none of the sets $R(A)$ and $R(B)$ is dense in $\mathbb{Q}_p$.

The first aim of the project *p-adic properties of certain combinatorial sequences and subsets of the set of non-negative integers* refers to obtaining a qualitative description of p-adic valuations of the numbers $H_d(n)$, where $p > d \geq 2$ are fixed values. Moreover, I intend to obtain results concerning combinatorial identities for the numbers $H_d(n)$, prime divisors of these numbers and periodicity of their remainders from division by a fixed positive integer. The second aim is to study p-adic denseness of ratio sets of values of quadratic forms and sums of fourth and fifth powers of non-negative integers. Additionally, I plan to consider ratio sets of sums of higher powers. Furthermore, I intend to answer the question if there exist two disjoint subsets of $\mathbb{N}$ such that their union is whole $\mathbb{N}$ and none of their ratio sets is dense in $\mathbb{Q}_p$ for any prime number p.