The current proposal is in the mainstream of algebraic geometry. Algebraic geometry is a part of mathematics that deals with solutions of systems of polynomial equations. In spite of seemingly narrow subject, in many cases it includes the study of much more general systems, e.g., given by holomorphic equations. The main motivations to study this subject are its applications in number theory, topology, differential equations and in mathematical physics.

One of the problems we plan to consider is question which holomorphic differential equations on complex manifolds come from geometry, i.e., which equations come from families of smooth projective manifolds. The holomorphic differential equations that we are interested in are classified by the associated representation of the topological fundamental group (so called monodromy). In some cases the space classifying such representations consists of a finite number of points and in these cases C. Simpson conjectured that the differential equations come from geometry. This was proven in the curve case by N. Katz. However, the corresponding open is almost completely open for higher dimensional varieties or in cases where the monodromy admits additional symmetries.

The remaining problems are more difficult to describe but all of them are related to the study of variation of holomorphic or algebraic structures on algebraic varieties. In some cases we plan to use non-commutative algebra even when studying commutative phenomena.

Even partial solutions of the considered problems should have important applications and woul allow to get a deeper understading of connections between differential equations, number theory, geometry and topology.