STOCHASTIC PARTICLE SYSTEMS WITH REPULSIVE FORCES
DESCRIPTION FOR THE GENERAL PUBLIC

INTRODUCTION

The search for universal patterns in very complex models is one of the most important task in mathematics. Such results help to describe the most important features of the considered phenomena using several selected properties. One of the most important examples is the Central Limit Theorem. It states that the sum of independent observations of random phenomena (e.g. the results of a dice roll, waiting times for a bus), after the appropriate normalization, approaches the normal distribution, when the number of observations increases to infinity. The obtained limit is always the same and the required normalization depends only on two characteristics: the mean and the variance. Other features do not matter. The independence of the components, which is the key assumption in the Central Limit Theorem, is very often unrealistic. In many important physical models we study strongly dependent objects. For example, the mathematical description of the phenomena of high energy physics is based on the so-called Hamiltonians. These are complex objects that are often modeled by random matrices, i.e. large square arrays of random numbers. The larger matrix we take, the better approximation we get. Energy levels in a given model correspond to the eigenvalues of the matrix. These expressions are strongly dependent, but some universal rules can be found, which yield their convergence, when the size of the matrix grows to infinity. It turns out that after appropriate rescaling, the limiting behaviour of eigenvalues (energy levels) is described by the so-called Wiegner's distribution. Also in this case, only few selected symmetry properties of the considered physical models and two characteristics of the random matrices matter. This claim was proved by Eugen Wiegner in 1955 and it underlies many important branches of theoretical and applied mathematics.

PROJECT OBJECTIVES

The aim of this project is to study the convergence questions associated with continuous-time analogues of random matrices. The considered random objects evolve (change) in time. Their eigenvalues are very important example of particle systems with repulsive forces. These are random objects that move in a chaotic manner, but as they approach each other they are repelled stronger and stronger. Repulsion of the particles is inversely proportional to the distance between them. We will study very general models of this type and look for answers to the question about their limit behaviour, when the number of particles increases to infinity. We will also be interested in particle collisions. It may happen that repulsive forces are weak and the chaotic movement of particles will lead to collisions. Examination of such systems becomes then very difficult, therefore it is important to provide necessary and sufficient conditions for no collisions. In addition, we will analyse the case of particle systems with weak repulsive forces, making suitable modifications to keep the original order. This may require additional reflection of particles at the moment of collision.

SIGNIFICANCE

This research project belongs to the probability theory and will be of significance to the theory of stochastic processes, in particular the theory of stochastic differential equations with singular coefficients. However, the context of the proposed research is much broader and it includes the theory of random matrices, non-commutative probability theory and applications in mathematical physics or statistical physics. The project continues the idea of generalizing Wigner's theorem to a wider class of matrices, which, in recent years, has been the driving force behind very intensive research in the theory of random matrices. On the other hand, the expected results will allow to define new classes of distributions and diffusions in free (non-commutative) probability.