Popular summary: Analytic Approach in Conformal Dynamics

Dynamical systems theory. Several problems are proposed for research which all belong to the area of dynamical systems theory. The theory historically grew out of celestial mechanics at the end of the 19th century and its founder is considered to be the Fench mathematician Henri Poincaré. He was the first prove that problems of celestial mechanics such as the description of motion of three gravitationally interacting bodies cannot be solved in a closed form. A need therefore arose to develop qualitative tools to describe various interesting aspects of motion.

As it turns out celestial mechanics is just an early example of mathematical modeling of change. By now such models are used in practically every branch of natural and social sciences. They usually cannot be solved in a closed form and most of the time are examined by computer simulation. Such simulation, however, could be meaningless if the system in question lacks stability and initial conditions are not chosen appropriately. Some level of qualitative understanding is therefore indispensable.

Research objectives. The first objective has to do with iterations of polynomials in the complex plane. A well-known object, even from popular science, is the connectedness locus or Mandelbrot set. Our detailed proposals concern the structure of the boundary of this set.

The second objective is about the metric attractors for a certain class of mappings of the circle and complex plane. A metric attractor describes the typical long-term behavior of orbits of the system which are chosen at random. The particular class of systems chosen here is noted for being difficult to study. The detailed goals in this are include computer-aided study.

The third objective is about the validity of computer simulations from the mathematical point of view. A barely correct program will give some numerical answer, but a mathematician must ask the question about the relation of this answer to reality.

Motivation. Progress in mathematics often occurs by studying concrete problems. The problem selected for this project are not new and known to be rather hard. Nevertheless, new and perhaps deep results are expected which will help refine the methods of dynamical systems theory. Such methods are needed to meet the needs of other sciences which use mathematical models.