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Dynamics from the single orbit point of view: quasicrystals, invariant measures, complexity

(Description for the general public)

Dynamical systems are models of phenomena evolving in time. Their analysis helps us to understand and predict the future of processes that are described via equations and functions.

This project focus on the interaction between global properties of a dynamical system and the behaviour of distinguished orbits. An *orbit* of a point is a set of states which given point visits according to the prescription for evolution characteristic for a given model. Realizing the project we will study these properties of dynamical systems that can be detect by observing the behaviour of a single point. This ,,single orbit" approach is implicit in a large part of topological dynamics. Roughly speaking, we try to answer the following questions: Given partial and/or approximate information about the orbit structure, how to recover complete orbits carrying the same information? How much knowledge of a system can be recovered from the existence of an individual orbit realizing that partial information? This is related to the properties of specification and shadowing. The former two notions are fundamental tools in topological dynamics. Shadowing means that approximate orbits known as pseudo-orbits (sequences of points where the next point is uniformly close to the image of the previous point through the acting map) remain close to real orbits, and specification allows us to approximate different finite pieces of genuine orbits by a single orbit. Both notions are associated with hyperbolicity, and both have a great many variants applicable to various classes of dynamical systems. Topological entropy measuring the exponential asymptotic growth rate of essentially distinguishable orbit segments of length n as n goes to infinity is in a natural way connected with single orbit approach.

We would like to analyze these orbits which give us the most information about given model, that is these whose behaviour is generic, typical for a given dynamical system. To be able to say what is generic we need a *measure* so that we know what is the statistical behaviour of an orbit in the model. We consider only *probability invariant* measures, that is measures which describe probability of some events in a dynamical system and such that these probabilities are not changing in time. Among them there are *ergodic measures*, that is measures with respect to which the given dynamical system is indecomposible into non-trivial subsystems. It is known that the family of all invariant measures has a structure of a simplex. In other words, it is similar to a multidimensional (maybe infinite dimensional) triangle. ,,The vertices" of this triangle are exactly the ergodic measures. The *Birkhoff ergodic theorem* says that if a measure is ergodic, then there exists a point such that every big set (with respect to this measure) will be often visited by it. What is more, there are many of them (with respect to this measure).

The study of invariant measures is helpful for understanding the properties of the dynamical system we examine. This can be illustrated by *entropy*. Generally, the more complicated is the dynamics, the bigger is the *topological entropy* of a given system. The *dynamical entropy* of an invariant measure neglects the dynamics outside the big set (with respect to this measure). It is known that the topological entropy is bounded from below by the dynamical entropy of every invariant measure and can be approximated by such entropies.

The class of zero entropy dynamical systems is rich. It contains for instance all isometries but also systems which are much less regular. Therefore to study the complexity of such systems we need more tools.

Realizing the project we will examine among others *quasi-cristals* which can be modelled as a space that is tilled with convex polyhedra in a regular but not periodic way (that satisfies some additional technical properties).