Every Banach space E is surrounded by a number of related metric and topological structures: one can discuss the weak topology on E, several topologies on the dual space E^*, properties of the natural embedding E into its second dual E^{**} etc. It is the interplay between metric and topological properties of those spaces that makes Banach space theory be such a fascinating area of study in pure mathematics.

There are close connections between Banach spaces and compact topological spaces. In one direction, every compactum K gives rise to a Banach space $C(K)$ of real-valued continuous functions on K; in the other, to every Banach space E one can associate a compact space $K = B_{E^*}$, the dual unit ball equipped with the *weak** topology, reflecting various properties of the space E. There is a well-studied class of Radon-Nikodym compacta; K is such a space if it can be embedded into some dual Banach space E^* having the so called Radon-Nikodym property. One of our problems is to describe compact linearly ordered spaces from that class. This seems to be closely related to studying set-theoretic properties of linear orders.

Another scientific objective of the present project is to settle some questions concerning the so called twisted sums of c_0 with other Banach spaces. Here c_0 is the classical space of sequences of real numbers converging to 0. This space has the following special property: Whenever c_0 is embedded into a separable Banach space E then it is complemented in E, i.e. there is a Banach space F such that $E = c_0 \oplus F$. There is a number of questions asking what happens when we embed c_0 into some nonseparable space E. For instance, assuming that c_0 is embedded into E so that the quotient space E/c_0 is isomorphic to $C(2^{\omega_1})$, where 2^{ω_1} is the Cantor cube of topological weight ω_1, one can ask: Must c_0 be compemented in E? We have recently proved that the answer is 'yes' in some model of set theory. It was already known that the answer 'no' is implied by the continuum hypothesis. Our argument made use of several auxiliary results on compact spaces and functions on Boolean algebras and we hope that, building on that idea, we shall answer several other questions in the topic and get more general results.

The measure algebra is a Boolean algebra obtained by taking Borel subsets in $[0, 1]$ and identifying pairs of sets that differ by Lebesgue negligible set. This is a classical object; on one hand, it is closely related to the Banach space $L_\infty[0, 1]$, on the other, it is used in the theory of forcing (under the name of random forcing) to prove consistency of several important statements. Many years ago M. Talagrand observed that several 'positive' results on integrating Banach-valued functions are true in the so called random model, while they do not hold if we assume more familiar axioms, such as the continuum hypothesis. We are planning to investigate properties of the measure algebra originating both in functional analysis and forcing.

Our research plan includes new methods of constructing Banach spaces. There is a new concept of considering, for a given Banach space E, the free Banach lattice $FBL[E]$ generated by E. The formal definition follows the usual pattern according to which free object are defined. The lattice $FBL[E]$ is essentially unique and can be realized in some space of continuous functions. However, not much is known about the structure of such lattices. We would like to understand, when $FBL[E]$ is weakly compactly generated, that is it contains a weakly compact subset generating it as a Banach space.

We plan to study systematically connections between Banach spaces and ideals on the set of natural numbers. An ideal on $\mathbb{N}$ is a hereditary family of its subsets containing finite sets and closed under finite unions. If $(x_n)_n$ is a sequence in a Banach space E then one can define an ideal of those $A \subseteq \mathbb{N}$ for which the series $\sum_{n\in A} x_n$ is unconditionaly convergent. Such an ideal belongs to the well-studied class of the so called analytic P-ideals. On the other hand, every analytic P-ideal can be associated to a subadditive function on $\mathbb{R}^{\mathbb{N}}$ which enables one to define certain Banach spaces. One of the objectives is to investigate Banach spaces obtained, in that way, from classical ideals on $\mathbb{N}$.