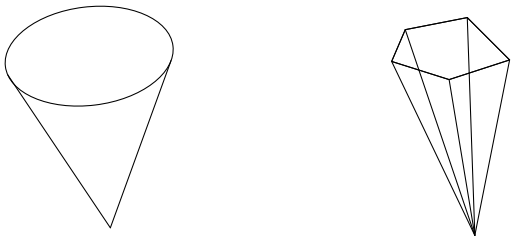
Reg. No: 2018/28/C/ST1/00339; Principal Investigator: dr Łucja Farnik

Positivity of line bundles on algebraic surfaces

Description of the project for the general public

The main objects of study in algebraic geometry are algebraic varieties. They are defined as the set of solutions of a collection of polynomial equations. In my study I investigate complex algebraic varieties, i.e., defined over the field of complex numbers.

Complex algebraic surfaces are varieties of dimension two. Among the most powerful tools in studying algebraic varieties are divisors. In case of surfaces divisors are a generalization of curves, one might say that they are linear combinations of curves with integer, rational or even real coefficients. Divisors with real coefficients form a real vector space of finite dimension. The dimension of this space is by itself a very interesting invariant of a surface. However, the divisors hold much more information about the surface. It can be extrapolated by analyzing the sets of divisors sharing certain properties, like positive intersection with all curves on the surface. For certain properties the sets of divisors having them are cones. One of the objects of my study is examining those cones, which will lead to a better understanding of algebraic surfaces. A priori the cones of divisors may have various shapes, e.g. may be rounded or polyhedral, see figure below:



The cones are infinite, however we can intersect them with a hyperplane given by a special divisor. Under some assumptions the resulting intersection will be bounded and its volume is an important invariant which will be investigated.

Apart from the cone volumes, Seshadri constants will be investigated in the project. The Seshadri constant measures how positive a divisor is. It is amazing that although the definition of the Seshadri constant is very simple, the exact values of these constants are very difficult to compute. Even the simple question whether Seshadri constants are always rational is still open.

I will also deal with k-jet ampleness of divisors on hyperelliptic surfaces. A question whether a divisor is k-jet ample is to ask whether the morphism assigning to a global section its values and derivatives up to order k_i at r points $x_1, \dots x_r$ is onto.

Most of the tasks of the project will be undertaken on hyperelliptic surfaces which are products of elliptic curves divided by a group action, but I will also work on other algebraic surfaces.

I will also study philosophical aspects of the conjectures connected to the 14th Hilbert's Problem which inspired the development of this area of algebraic geometry.