

Solving the paradoxes of naïve validity and informal provability through non-deterministic and truth-maker semantics

Paweł Pawłowski

From a broad philosophical perspective this project is located between the epistemology of mathematics and the part of analytic and formal philosophy devoted to semantical paradoxes. Two such paradoxes are relevant for my project: the paradox of naïve validity and the paradox of informal provability. Roughly speaking, they show that it is not possible to represent these notions in the object language of a very natural formal system— formal arithmetic. To obtain the paradox, one has to construct a certain self-referential sentence, which together with intuitive principles for either informal provability or naïve validity allow one to infer a contradiction within the system.

Representing informal notions that are central for the epistemology of mathematics is philosophically important and challenging. On one hand, these attempts show to what extent formal systems can talk about their own semantical properties. It is very interesting to see what a formal system “sees” as valid within the object language. On the other hand, this not only provides a deep insight into semantical notions, but also may show the scope and the limit of formal methods in this particular context.

In this project I will study the notion of naïve validity. Roughly speaking, an argument is naively valid if the truth of its premises implies or guarantees the truth of its intended conclusion, given what the sentences involved mean. The main problem with this notion is that as soon as we employ formal methods in order to represent it in the object language, a paradox involving a self-referential sentence arises. To solve the paradox, I will look at naïve validity as a generalization of the notion of informal provability. An informal proof is a commonly accepted mathematical justification of a mathematical claim. The ultimate goal of my research is to overcome the paradox by constructing a formal theory based on a philosophically interesting framework of informal provability— non-deterministic approach to informal provability. On this approach, some mathematical sentences are informally provable, some are informally refutable, and some are neither. In order to model this formally non-deterministic logics BAT and CABAT were developed. The lack of truth-functionality stems from the observation that disjunctions and conjunctions of sentences which are neither informally provable nor refutable may have a different status depending on the relations between these sentences. Unfortunately, the BAT framework is not fully developed. Thus, the main technical aim is to generalize and adapt the non-deterministic framework of informal provability to naïve validity. The main philosophical task is to interpret the formal results and to explain the relation between naïve validity and informal provability.