DESCRIPTION FOR THE GENERAL PUBLIC

The main scientific objective of the project is to verify the hypothesis according to which the existence of a multi-frequency (i.e. N-frequency, where $N \geq 3$) quasi-periodic solutions is widespread in closed arrays (rings) of coupled nonlinear oscillators and also to identify the mechanism and conditions of a dynamic process, which must be satisfied for this solution occurred.

A typical dynamic effect observed in these systems of many oscillators, coupled unidirectionally in a closed loop, is the occurrence of so-called rotating wave wandering along this loop. According to the adopted hypothesis synchronization of the rotating wave with individual responses of node oscillators and the correlation with other independent rotating waves, in the case of complex configurations of unidirectional coupling, can lead to the appearance of the stable N-frequency quasi-periodic solutions (N-dimensional tori) as an effect of the subsequent Hopf bifurcations occurring as a result of changes of the overall coupling parameter. First time such a scenario of dimensionally increasing quasiperiodicity has been proposed by Landau and Hopf as an explanation of transition to the turbulence. However, later work by Newhouse, Ruelle and Takens had demonstrated that just after third step of this bifurcational scenario there appear chaotic strange attractor as an effect of arbitrarily small perturbation of the 3D torus (NRT scenario). Validity of this scenario has been verified in large family of dynamical systems. . On the other hand, other researchers shown numerical, experimental and also analytical results, which confirm the possibility of the stable 3D-torus existence.

In general, the concept of research provides the following stages of the project implementation which can be treated as project tasks:

- Development of series of numerical models of coupled oscillators with varied complexity (simple loops, less or more complex connections – see Fig. 1) of unidirectional coupling in two variants:
  - continuous time systems (phase streams – phase, Duffing or Van der Pol oscillators, Lorenz or Rossler systems),
  - discrete time systems (maps – logistic, circle or Henon map).
- Bifurcation study of developed models, supported by the calculation of Lyapunov exponents and the FFT analysis, where overall or local coupling strength is a control parameter.
- Detailed numerical and theoretical analysis of possibly identified multi-dimensional quasiperiodic solutions.
- Construction of the experimental rig (electrical circuit) for the most interesting of numerically detected cases of high-dimensional quasiperiodicity.
- Experimental verification of numerical results and analysis of the parameter's mismatch influence on the stability of observed solutions.

Summing up, the reason for taking this research subject is to confirm the adopted research hypothesis that it is possible the realization of classical scenario of transition to turbulence/chaos i.e., via series of subsequent (at least three or more) Hopf bifurcations.