

For more than four decades, there is a growing interest of engineers on fractional order derivatives and differences. Modern materials used in electrical engineering, mechanics, and other fields of technology require new mathematical operators to describe their properties that take into account, for example the memory of the past states in these materials. In addition, for many dynamic systems, the description using classical differential or difference equations of integer order does not provide sufficient precision - especially at high requirements for the quality of control of such objects.

In recent years, many research centres in Poland and abroad have undertaken the studies on differential and difference calculus of arbitrary orders. Various definitions of fractional order derivatives have been formulated, the most popular are the derivatives of Caputo and Riemann-Liouville in the case of the continuous-time functions and the Grunwald-Letnikov definition of the fractional order for discrete-time functions. These definitions preserve many of the properties known from the differential and difference calculus of integer orders, although the properties such as differentiation of a constant function, chain rule, differentiation of product or quotient of two functions are not generally satisfied. In addition, there are many problems with the interpretation of the initial conditions for this type of definition.

This has become the motivation for finding some new definitions of fractional order derivatives with properties that are compatible with the classical differential calculus. In recent years two interesting definitions of fractional order derivatives have been introduced: Caputo-Fabrizio and CFD (Conformable Fractional Derivative). The first one solved the problem of the singularity of the kernel of Riemann-Liouville and Caputo operators. CFD definition satisfies all the properties well-known for classical differential calculus including Leibniz rule, differentiation of product and quotient of two functions. This makes these two new operators a very useful tool for real dynamic objects modelling.

The aim of the project will be a comparative analysis of dynamic systems described by differential equations of fractional orders using different definitions of fractional order derivatives. One-dimensional and two-dimensional systems described by fractional order state equations will be considered. Moreover the systems with different orders of fractional derivative will be analyzed as well as the systems with nonlinearity in the state space equation. Additionally the subject of the research will be the systems with parameters depending on time and the systems which parameters values are changing in some defined interval.

For the aforementioned systems, classical control theory problems will be analyzed. For each definition a solution of the fractional order state equation will be formulated. The conditions for positivity, asymptotic stability, reachability, controllability to zero will be analyzed. For systems with uncertain parameters, well-known from classical control theory, the Charitonov theorem will be generalized for fractional order systems. Similarly, for nonlinear systems, the Popov theorem will be used to formulate the stability conditions.

The results of the analysis will be supported by numerical analysis and simulations in the Matlab computational environment. Next, after the verification of the usefulness of the obtained results, they will be confirmed by real objects measurements. The control algorithms developed in the project will be implemented in a microprocessor system and tested using the PXI computer.

The project will extend the theoretical knowledge in fractional order systems which is an essential part of a modern control theory and systems.