

DESCRIPTION FOR THE GENERAL PUBLIC

This research project concerns important aspects of analysis related to radially symmetric objects (invariant under rotations around the “center”), associated maximal operators, and partially to non-trigonometric orthogonal expansions.

One of the research tasks pertains to the long-standing open problem of finding a precise description of the behavior of the heat kernel on a sphere of arbitrary dimension, in particular on the “ordinary” sphere of dimension 2. This is an integral kernel of the spherical heat semigroup, which solves the spherical heat equation, and thus describes propagation of heat in such media. Simultaneously, the kernel is a transition probability density for the spherical Brownian motion, an important stochastic process modeling motion of a diffusing particle on the sphere. Clearly, these two facts lead also to physical significance of the spherical heat kernel.

Another research task aims at studying maximal operators related to averaging operators over spheres (the so-called spherical means or spherical Radon transform) and well motivated extensions of these operators. Spherical means and their generalizations are of great importance in analysis because they constitute a theoretical background to a number of physical and practical applications such as, for instance, thermoacoustic and photoacoustic tomography. They also describe solutions of several classical initial-value partial differential equation problems, including the wave equation, the Tricomi equation, or more generally the Euler-Poisson-Darboux equation. The principal motivation to study the above mentioned maximal operators is a fundamental question about satisfactory connection of these solutions to (possibly irregular) initial data.

The project embraces also investigations of maximal operators associated with discrete variants of the spherical means on spaces having lattice structure (for instance, a finite product of integers). Such operators are not well understood yet, because of serious technical difficulties occurring in their treatment. However, they are very intriguing and of great importance, especially due to their deep connections with the number theory and the ergodic theory.

The remaining research tasks concern, among other things, non-trigonometric orthogonal expansions (which are generalizations, in various directions, of the classical Fourier series) and the Dunkl analysis. The project includes analysis associated with classical orthogonal expansions such as Fourier-Bessel, Laguerre and Jacobi expansions. They exist in the literature for a long time, their study being motivated similarly as in the classical situation, and recently one could observe a revival of interest connected with their study. Within the project tasks a measurable extension of the present knowledge in this area is planned. On the other hand, the Dunkl theory is a young, but dynamically developing mathematical concept with interesting physical interpretations in the background. It is, in a sense, a perturbation of the classical situation related to the Fourier transform by reflections with respect to systems of hyperplanes (“mirrors”). The objective of one of the research tasks is to reveal that certain Dunkl contexts are reasonable harmonic analysis environments to much larger extent comparing to what was believed before.

The results obtained within this project would be new, original and would constitute a noticeable contribution to real and harmonic analysis, especially to the subarea related to orthogonal expansions. They would deliver answers to both interesting long-standing open questions and those which arose only recently, in connection with a dynamic development of this area of analysis. The research proposed in this project would also require original ideas and essentially new techniques that would be of interest in their own right due to their potential applications in a future research.