DESCRIPTION FOR THE GENERAL PUBLIC

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The calculus of variations is a branch of mathematics, whose goal is to find the minima and maxima of functions defined on infinite-dimensional objects; in most cases these objects are function spaces. It was created in XVII-XVIII century and the first problems from this field were the problem of finding a curve of fastest descent and connecting minimalization problems with partial differential equations using Euler-Lagrange equations.

In every variational problem there are three most important issues: existence of solutions in a properly chosen class; regularity of solutions, i.e. if the solutions have any additional properties; finally, the uniqueness of solutions. We are going to inspect each of these issues for the *anisotropic least gradient problem*. We are motivated by the *conductivity imaging problem*, where solving an anisotropic least gradient problem appears in an algorithm designed to calculate the conductivity inside Ω from a single measurement of current density inside Ω and the voltage on the boundary of Ω. Furthermore, this problem is related to the *free material design*, which concerns the optimal design of the shape of a conducting medium.

The anisotropic least gradient problem is the problem of minimalization of the total variation of a function on a given domain Ω with respect to the boundary data f defined on the boundary of Ω. The problem is anisotropic, i.e. the total variation of a function is not measured in the Euclidean norm, but in some given norm ϕ, which may depend on the direction of the derivative of the function and on the location inside Ω. The goal of this project is to investigate the role played by the geometry of the domain Ω and the regularity of ϕ in the anisotropic least gradient problem. In this way, we want to bridge the gap between the two main ways in which this problem was considered: the first one used the methods of geometric measure theory, but strictly relied on continuity of the boundary data, while the second was a more general approach which allowed for discontinuities, but required a more lax interpretation of the boundary condition.

We want to discuss three issues: firstly, the existence of solutions for a sufficiently large class of discontinuous boundary data. Secondly, we want to study the issue of uniqueness of solutions for discontinuous boundary data; while we do not expect the minimizers to be unique, we expect that for sufficiently regular ϕ they share an almost identical structure of level sets. In both of the above cases we want to weaken the geometric assumptions on Ω as much as possible. Finally, we want to study the uniqueness of solutions for continuous boundary data, but without any regularity assumptions on ϕ. Here, we will study two cases, whether the unit ball in the dual norm ϕ^* has flat parts or not. If there are no flat parts, we expect the solutions to be unique for any continuous boundary data. In particular, a result of this kind would free the issue of uniqueness from assumptions on the regularity of ϕ and enable us to approximate the solutions numerically by adding arbitrarily small regularizing terms.