REGULARITY AND ASYMPTOTICS FOR MULTIDIMENSIONAL MARKOVIAN EVOLUTIONS

DESCRIPTION FOR THE GENERAL PUBLIC

Introduction: Many physical, biological and social phenomena can be modelled as evolution of a particle. By evolution we mean a Markovian mechanism, i.e. one in which the future configuration, or position, depends on the present and does not depend otherwise on past configurations. We will deal with multidimensional phenomena in the sense that a particle may be located in a space of arbitrary dimension. As proper for the mathematical model, we attempt to capture the fundamental features of the dynamics, thus resolve the critical difficulties by proposing an abstract but flexible mathematical framework offering a simplifying perspective for complicated phenomena. One source of the simplification is the correct mathematical language and the other, more important source of simplification is that complicated Markovian systems often have a limiting or stationary behaviour in large time. We will use probabilistic, potential-theoretic and analytic methods to obtain such results.

Project objectives. We want to prove Harnack's inequality for harmonic functions with respect to jump Markov processes. This inequality is an important tool in investigations of properties of harmonic functions. It was used in many important classical theorems such as Liouville's theorem, the removable singularity theorem and Harnack's first and second convergence theorems. We will also prove Hölder continuity for harmonic functions. We will further study the quasi-stationary measures, i.e. initial distributions of evolutions which, in a sense, do not change over time. Such distributions often come up as normalized limits of the considered processes and consequently they are very important in studying the asymptotics behavior of the evolutions.

Significance. This research project belongs to the theory of stochastic processes and the main focus is on asymptotic properties of the multidimensional Markov processes and the underlying Markovian evolutions. However, because of numerous and deep links with other areas of mathematics, the outcome of the project will also be significant for the potential theory and the theory of non-local partial differential equations. Selected results may also find practical applications, since the considered class of processes includes many examples appearing in mathematical physics and in the statistical and financial modelling.