DESCRIPTION FOR THE GENERAL PUBLIC

Research project objectives/Research hypothesis

In the project some chosen fine-grained measures of “theory strength” are applied in order to investigate the strength of axiomatic theories of truth.

One of the main objects of study in logic are theories and sentences treated just as strings of characters: by "theory", we can understand, for instance, all sentences printed in a given book. Sentences conceived in such a way have various properties, just as all other objects. Some of these properties clearly admit precise definitions. In order to define such predicates like "*x* consists of at least 17 letters" or "*x* contains parentheses", we only need to describe shapes of sentences in a precise manner.

The famous theorem by Tarski states that the *truth* of sentences is not one of those properties which allow a precise definition. As long as we are using the classical logic, we cannot precisely state what we mean in general by saying that a sentence is true without running into inconsistencies. It is not possible to define a property P such that the sentence "Snow is white" has the property P, the sentence "Tarski was a logician" also has the property P, but the sentence "Snow is green" does not have that property, with this pattern repeating itself for all sentences (including the ones containing the truth predicate). Tarski's theorem describes our fundamental cognitive limitations and as such, it is obviously philosophically interesting.

However, if there is no predicate satisfying all the conditions which intuitively correspond to the property of being true, then a question arises concerning the existence of consistent predicates whose behaviour *partially* resembles that of the truth predicate (truth-like predicates). For instance, it turns out that we can define properties which hold of a conjunction (A and B) if and only if they hold of both conjuncts separately. The list of conditions which can be demanded from predicates defined in a precise manner without running into inconsistencies is significantly longer than that. The predicates whose properties resemble those of the truth predicate are investigated in *formal truth theory*, a line of research of which our project is a part.

One of the basic relations between theories describing truth-like predicates is revealed by the comparison of the sets of those consequences of the theories which can be expressed without any semantic notions (like truth, reference, or meaning). In other words, we compare then those consequences which can be expressed without any use of the truth predicate (consequences formulated "in the base language"). This corresponds to investigating which assumptions about the truth predicate carry some information about the world, and which of the axioms cannot be used to draw any conclusions concerning non-semantic facts, even though they provide a refined description of the behaviour of the truth predicate.

However, it happens rather often that very different assumptions about the considered predicates yield exactly the same consequences in the language of base theory. Then we have to turn to more sophisticated methods of comparing such assumptions.

One of the methods of comparing two systems of axioms consists in examining whether it is possible to define a predicate satisfying the first set of axioms in terms of a predicate satisfying the other set of axioms. If we can produce such a definition, then, in a sense, everything that we can expressed with the predicate satisfying the former set of assumptions can be expressed using a predicate satisfying the latter system of axioms. This is one of the methods (but not the only one) of comparing truth axioms which are subtler than simply comparing sets of consequences. Our project is devoted to investigating these fine-grained relations between various important axioms describing the behaviour of the truth predicate.

Research project methodology

In our project, we would like to primarily use formal methods, together with an analysis of the philosophical notions with which our research is concerned, so that, in the end, we can assess the viability of various assumptions about the truth predicate.

Expected impact of the research project on the development of science

Our research is fundamental with no intended direct applications. However, achieving a better understanding of the properties of the truth predicate may help us to shed light on the philosophical puzzles concerning this notion. Philosophical discussions are already making heavy use of the findings on formal truth theories and, conversely, they also stimulate formal research via asking some crucial questions. The results on formal truth theories have also been used in the research on foundations of mathematics, hence the project has consequences also for this area.