Description for the general public

In the project we will consider models of inelastic deformation. We assumed that considered material, with the mass density $\rho > 0$, lies within the three dimensional set Ω. If we apply a force to the body (for example by squeezing it) it should cause some displacement in the material. The displacement are described in every point x of the set Ω in every time $t \geqslant 0$ by the three dimensional vector $u(x,t)$. Deformation occurring in the material can be described by so-called small deformation tensor $\varepsilon(x,t)$ which is computed based on the displacement. Deformation $\varepsilon(x,t)$ can be divided into the elastic part (reversible when the forces are no longer applied) and the plastic one (irreversible) in the following way

$$\varepsilon = \underbrace{\varepsilon - \varepsilon^p}_{\text{elastic}} + \underbrace{\varepsilon^p}_{\text{plastic}}.$$

To be reversible the elastic deformation have to generate some stress in the material. The stress in the material is described by so-called Cauchy stress tensor $\sigma(x,t)$. Hooke's law state that the elastic part of deformation is proportional to the Cauchy stress tensor. Using Newton's second law one can derive equations which connect displacement (x,t), deformation tensor $\varepsilon(x,t)$ and stress tensor $\sigma(x,t)$.

$$\begin{aligned} \rho\, u_{tt}(x,t) - \operatorname{div}_x \sigma(x,t) = F(x,t)\,, \\ \sigma(x,t) = \mathcal{D}\left(\varepsilon(x,t) - \varepsilon^p(x,t)\right), \end{aligned}$$

where $F(x,t)$ describes a density of applied body forces, $\mathcal{D}$ is so-called compliance tensor (it depends on considered material). Our goal is to find unknown functions: $u(x,t)$, $\sigma(x,t)$ and $\varepsilon^p(x,t)$. Unfortunately, we have three unknown functions and only two equations. The third equation is experimental. Often it connects $\varepsilon^p(x,t)$ with $\sigma(x,t)$ and is given in the following form

$$\epsilon^p_t(x,t) \in \mathcal{G}(\sigma(x,t)).$$

Such equation is called the constitutive equation and the function $\mathcal{G}$ is called the constitutive function.

Our goal is to investigate such models and proving that for important constitutive function and under other reasonably easy to check assumption it has a solution. In particular (but not only) we are interested in Prandtl and Reuss constitutive equation (the model of ideal plasticity). Such constitutive function has been proposed in the 30s of the XX century and only known results (obtained in the 80s) claim that there exists a solution under the specific and difficult to check assumption called *the safe-load condition*. Our main aim is to replace the safe-load condition by much easier to check assumption about the set of admissible stresses. Therefore, it will be much easier to verify if a specific model can be used in the given situation (known material and applied forces). What is more, we would like to show the existence of more regular solution than known in literature. Such results will significantly increase the knowledge about plasticity and moreover, it should make easier using the plasticity models in practice.