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Complex contact manifolds and geometry of secants
Description for the general public

In Mathematics, the purpose of *classification theorems* is to divide objects of interest into several groups, according to a specific key. Analogously, in other sciences we also *classify*. For instance, organisms are divided into six kingdoms, including bacteria, plants, animals, fungi,... A mathematician, if he is not 100% sure of his classification, may talk about a *conjecture*. This happens if we fail to find an example, which does not match into the classification scheme, and at the same time we cannot prove that the classification is complete.

In the project we address two major problems that combine the methods of Algebraic Geometry with Differential Geometry, Partial Differential Equations, Algebra, Representation Theory, Topology, and Computational Complexity. The first problem is the classification of complex contact Fano manifolds, known as LeBrun-Salamaon Conjecture. The second problem is related to the properties of secant varieties and the notions of tensor rank or Waring rank. It is concerned with Conjectures of Comon and Strassen.

In a significant simplification, a classification of contact manifolds leads to a classification of other mathematical objects, namely, the quaternion-Kähler manifolds. These describe one of the building blocks for any Riemannian manifold, that is a set with smooth structure and a metric, so that, informally, we can express what has corners, what is smooth, what are the angles, and how far away are the points. Thus both a proof or a conterexample to LeBrun-Salamon Conjecture would lead to a better understanding of the structure used to build any such geometric set.

For many sciences it is critical to extract simple and meaningful ingredients from some complicated data. In mathematical terms, this corresponds to the problem of decomposing tensors. The key notions here are *rank* and *secant variety*. As an example, imagine a single antennae that receives a signal from many mobile phones at the same time. The receiver must decompose this superposed electromagnetic wave into original *simple* signals, each one encoding a single conversation. As another example, fluorescence spectroscopy is a method to analyse samples of solutions and determine concentration of chemicals. Each sample is excited by light at various wavelengths and the light emitted is measured. The data is collected as a tensor, and we need to determine the decomposition of this tensor in order to extract information about the chemicals and their concentrations. The notions of simple ingredients, that we want to decompose into, depend on the particular problem. Sometimes it happens that for a given mathematical object we can calculate its complexity in two diffrent ways. Conjectures of Comon and Strassen deal exactly with such situations, and they predict that the complexity (or rank) is the same, independent of the method.