Nonlinear equations involving the curl-curl operator

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The research project is devoted to study nonlinear curl-curl problems arising in electromagnetism. Firstly, we look for solutions of time-harmonic Maxwell equations in the presence of nonlinear polarization which form models widely investigated in physics and engineering. Nonlinear materials, for instance Kerr-like media, materials with saturation or cubic-quintic effects, play a crucial role in nanotechnology and allow to consider physical structures which are often smaller than the wave length of light. Such nonlinear structures have new and fascinating optical properties that cannot be modelled by using their linear counterparts. The aim of the project is to study time-harmonic electromagnetic waves in nonlinear media by means of variational and bifurcation theory. The usage of these methods is novel for the nonlinear curl-curl problems and require development of new analytical techniques. We investigate the existence of ground state and bound state solutions of the semilinear Maxwell equations under the effect of a general nonlinearity, their multiplicity and symmetric properties. Moreover we are interested in semiclassical states, normalized solutions as well as systems of equations involving the curl-curl operator.

Secondly, we want to study curl-curl problems arising in the Born-Infeld theory. Recall that in the 1930's, Born and Infeld constructed a new theory of the electromagnetic field by introducing a nonlinear Lagrangian density. From the mathematical point of view, the Euler-Lagrange equations in this theory are very challenging and there is very little work on the problem. Nowadays, the methods of nonlinear analysis seems to be sufficiently strong to provide rigorous analysis of the Born-Infel theory. Indeed, very recently electrostatic solutions have been investigated and our aim is to analyse the magnetostatic solutions, where the nonlinear curl-curl operator appears.

The research program lies at the borderline of the following fields: variational methods, bifurcation theory, partial differential equations, functional analysis and mathematical physics. The potential applications of the expected results may result in better understanding of the physical models, e.g. of nonlinear optics or of the Born-Infel theory, and in obtaining more efficient numerical methods for the curl-curl equations. As a potential output, we expect that the existence of solutions to our research problems and their multiplicity will be obtained. Moreover we want to employ the bifurcation theory for the nonlinear curl-curl problems, which has not been used in this context so far. We intend to develop new mathematical techniques allowing to study also other nonlinear partial differential equations like nonlinear wave equations or Schrödinger equations. Standard variational and bifurcation methods seem to be not applicable due to the presence of the strong indefiniteness of the problems. New techniques are required which will be of interest for specialists in all the above fields. Moreover the expected results will give raise to the further studies of the dynamics of the time-dependent nonlinear electromagnetic wave equation in the spirit of pioneering works of Cazenave, Kenig, Merle and Tao devoted to nonlinear Schrödinger and wave equations.