One may say that all algebraic varieties are in between the worlds of continuous (geometry) and discrete (algebra) objects. They may be investigated from both points of view, but using both approaches usually gives the best effect. In the present project we investigate varieties which lie a bit more on the discrete side – they are related in a natural way to certain combinatorial objects, and applying discrete and computational methods to them allows to obtain a lot of information. We have chosen three topics with this property.

**Cox rings of resolutions of quotient singularities.** The Cox ring is an invariant of a variety which contains a lot of information on its geometric properties, and also on the geometry of its small modifications, i.e. varieties very similar to it. We investigate Cox rings of resolutions, that is smoothings, of quotient singularities – spaces of the form of a quotient $\mathbb{C}^n/G$, where $G$ is a finite group. From the structure of the Cox ring we would like to obtain information on crepant, i.e. minimal in some sense, resolutions of quotient singularities. In this project we will look for small generating sets of these rings – based on our earlier results we expect that such sets should have a good combinatorial description. Small generating sets may be used, in particular, for understanding the cone of movable divisors of a resolution and its decomposition into smaller cones, the Mori chambers. This combinatorial structure describes the relation between small modifications of a chosen resolutions, and in dimension 3, which is one of the topics of this project, even the relation between all crepant resolutions of a chosen singularity. The results obtained this way will be a significant contribution to the area of resolutions of quotient singularities. There are little new results recently in this area, and before the methods related to Cox rings were applied to this problem, even the numbers of all crepant resolutions for some of the smallest examples of groups were not known.

**Tropicaliztion with additional structure.** Tropicalization is a method of assigning a fan in a real vector space to algebraic varieties, based on a valuation on the coefficient field. We are planning to investigate two problems related to this construction. The first one is an intriguing open question from a paper by Sturmfels and Xu; it concerns classification of specific generating sets, known as Khovanski bases, of the Cox ring of del Pezzo surface of degree 3. Although the tropicalization does not appear in the formulation of the problem, we expect that it is the key element of the solution – the structure which parametrizes Khovanskii bases. The second problem concerns very basic properties of the tropicalization: for a variety with a group action we want to investigate the induced action on the tropicalization and its relation to the original action. This is a very broad topic and any partial result will be important for understanding the operation of tropicalization.

**Varieties related to matrices, torus actions and p-divisors.** The last topic concerns p-divisors – partially geometric and partially combinatorial structures introduced by Altmann and Hausen for describing varieties with an action of a torus, whose maximal orbit has dimension smaller than the variety. A p-divisor consists of geometric objects (divisors), to which we assign coefficients which are polyhedra with vertices in a certain lattice related to the acting torus. Up to now, not many examples of p-divisors have been described in the literature – the existing ones belong to two classes. We are planning to look for new examples among varieties which can be defined with the use of a (permutation) matrix or a sequence of matrix groups: matrix Schubert varieties, Kazhdan-Lusztig varieties and Bott-Samelson varieties. We expect that the combinatorics of polyhedral coefficients can be expressed in terms of matrices (or matrix groups) determining the variety. These results should provide a lot of examples of p-divisors, different from the ones already described. Moreover, we hope that investigating a large group of such examples may initiate a new project in this area – either applying combinatorial methods for a better understanding of p-divisors on using p-divisors in problems which are closer to combinatorics.