

Let's imagine a wizard's hat lying down on the table. Its surface, looking like a cone, may be viewed as a graph of certain continuous function over a disc on the plane (let us denote this plane by \mathbb{C} - this gives us some additional information about the complex structure we introduced there, but for the moment it is not important to us). We see that at the point, above which the vertex of our cone is (we assume here that the wizard cares about his hat and never allows it to break) the value of our function is the biggest - it attains its maximum there. Imagine further, that, with a huge risk of awaking a wizard's anger and being transformed into a frog, we have cut out the top of the hat and then we have stitched a patch on the hole - with no spike this time. If we were lucky (and first of all we haven't been turned into a frog) the surface we have just produced looks, at least observed from some distance, like a graph of some differentiable function and it still attains some maximum at the point from the interior of the disc. This kind of phenomenon would not be possible, if the function under our consideration were the absolute value of some holomorphic mapping, which means complex differentiable. This is due to the so called maximum principle, which says that the module of the non-constant holomorphic function cannot be attained (even locally) in the interior of the domain, where the function is defined. If now our non-constant holomorphic function is assumed to be additionally continuous up to the circle bounding our disc, then it only can take its maximum at that circle. Actually, the disc may be replaced here by any bounded domain, and the circle - by the boundary of such domain. An interesting question is the following opposite problem: given a boundary point ζ of a bounded domain, can we construct a holomorphic function in a neighbourhood of our domain, such that its module, when restricted to the closure of the domain, attains its maximum at that point? This kind of function is called a peak function at ζ . For the disc discussed above the answer is: yes (try to do this - it's not very complicated!) More generally, we can always find such function, whenever our domain is strictly pseudoconvex (the simplest example of such domain is... a disc on the plane), not only on the plane, but in any complex dimension - in every \mathbb{C}^n !

In our research we shall investigate the possibility of smooth (continuous) transforming such peak functions one into another, once we are given a family of strictly pseudoconvex domains varying smoothly. The similar question we are interested in, concerns the same problem for the families of the so called exposing functions at the boundary points ζ , that is, the holomorphic embeddings of the domain with the property that the image of ζ is a point of global strong convexity (for example, each boundary point of the disc is such a point).

Those issues are important, since the objects we are going to focus on, are useful tools for mathematicians working in the areas of complex (and functional) analysis, and they sometimes allow those scientists to write the beautiful masterpieces - mathematical theorems should be considered as those.