Deformations and Degenerations of Algebraic Varieties

Description for the general public

Piotr Achinger
Instytut Matematyczny Polskiej Akademii Nauk

The proposed research belongs to algebraic geometry, with applications to arithmetic. Algebraic geometry studies sets of solutions of systems of polynomial equations, called algebraic varieties. The classical theory concentrates on varieties over the field of complex numbers, but from the point of view of number theory and arithmetic, algebraic varieties over the field of rational numbers and over finite fields are the most important. In our project, we plan to study both complex varieties and those in positive characteristic, bringing together techniques of topology and algebraic geometry.

The goal of the project is to study *families* of algebraic varieties, which in simple words are variations of algebraic varieties depending on a parameter t. The utility of families can be seen from various points of view:

- Given a single geometric object, one can degenerate it to a special fiber. This operation tends to destroy some of the structure, and the degenerate member of the family can be easier to study than the given object. For example, a triangle can be degenerated into an interval.
- Using the degeneration procedure described above one can turn a geometric object (a variety over the complex numbers) into an arithmetic object (a variety over a finite field) and study it using the methods of arithmetic geometry.
- Given an object X_0, one can try to find a family with X_0 as a special fiber. Finding such a family (usually non-unique) allows one to apply e.g. the methods of mathematical analysis. Such a family does not always exist, and the obstructions to its existence can give insight about the pathologies of X_0.

The more specific goals of the project are briefly sketched below.

Logarithmic geometry is an essential tool in the study of degenerating families. We plan to use it to better understand the topology of algebraic maps.

Deformation theory in positive characteristic. Calabi–Yau varieties serve as models of hidden spacetime dimensions in string theory, and mirror symmetry is a conjectural but mathematically precise dualism between such varieties. In order to apply ideas of mirror symmetry in arithmetic geometry, one needs to understand deformations of such varieties over finite fields and when such varieties “come” from characteristic zero. In a different direction, we aim to construct **new non-liftable schemes**, which are geometri cobjects which “live” only in positive characteristic.