Popular description of the project 'Measure theoretic, geometric and fourier methods of analysis...'

The aim of our studies is analysis of some concrete solutions of 2d Euler equations and the Camassa-Holm equation.

On the one hand we are going to examine the spiral vortex sheets as irregular solutions to the 2d Euler equations. Such spirals play an extremely important role in understanding the flying phenomenon. Let us assume that Euler equations describe the motion of the air in a satisfactory way. In view of d'Alembert's paradox as well as the Helmholtz theorem, sarting from zero-vorticity initial condition, the inviscid regular flow of air does not exert any lift on the wing of a plane. However, it was observed that spiral-shaped irregular solutions occur. Physicists as well as engineers, at least since the beginning of XXth century and the school of Prandtl, were associating such objects with the lift phenomenons. Under the latter perspective the ability of computing such solutions, in particular finding accurate numerical schemes, as well as studying their properties, are the tasks of extreme importance. Many studies were related to such objects. Unfortunately the satisfactory theory of spiral vortex sheets as solutions of 2d Euler equations is still missing. It is one of our goals in the project to work out mathematical fundaments which would help to extend our understanding of the way in which such objects could be interpreted as solutions of 2d Euler. When pursuing such issue we immediately meet essential obstacles. First of all one needs to define a sense in which such irregular spirals could be interpreted as solutions of differential equations. Next question is that we cannot be sure that Kaden's or Prandtl's spirals, best known models in the literature, are indeed solutions of homogeneous Euler equation. Perhaps one needs to identify some perturbations of Euler equations being satisfied by our objects of interest. A natural point of view on vortex spirals is viewing them as σ-finite measures supported on the spiral. One of the problems related to such an option is that we have to deal with measures which are not compactly supported. That requires some new measure theoretic ideas. Even identifying divergence-free velocity generated by such spiral of vorticity of unbounded support does not seem trivial. It seems to us, in view of our recent results, that so called complex moments of measures would play an important role in further studies. Next problem we need to face when building our theory is the problem of uniqueness of solutions in the class of objects including our non-compactly supported measures. Indeed, we would like to make sure that numerical scheme approximations of solutions do not converge to some other objects. The latter uniqueness problem is close to a long-standing open problem of uniqueness of Delort's solutions.

The second main goal of our project are studies of the evolution of so called multipeakons, solutions to the Camassa-Holm equation. Camassa-Holm is a model of motion of water in a shallow channel. It captures the wave breaking phenomenon. One of our tasks is to find necessary and sufficient condition for the initial configuration of the multipeakon distinguishing between wave collaps or not. In our studies we shall use the Hamilton equations (and geometric methods related to them) describing the evolution of multipeakons. However one would also like to understand the evolution of the multipeakon after the wave collapses. But this requires studies of irregular solutions. Further studies of the evolution require extension of differential geometry methods to such a singular case. It seems to us that the concept of viscosity solutions and the connection between Hamiltonian dynamics and Hamilton-Jacobi's equations could be of importance.