

Nowadays, computer simulations are commonly used for verification of many engineering problems. For each new design of a car, there are car crash simulations performed on computers, to verify the strength of the car construction during car accidents, when the car is hit from different directions, with different hitting force. Similarly, for each new design of an airplane, there are some computer simulations performed, concerning its strength as well as the possibility of vibration of the aircraft construction during take-off, flight, and landing. Additionally, in the field of civil engineering, each new structure has to be verified by performing computer simulations of the stress distribution over the construction and its strength under mass forces, forces generated by wind, snow and sand storms (in some geographic areas). Computer simulations have also some applications in medicine, for example blood flow modeling in central arterial system is used to predict the blood flow through bypass, or the behavior of the aortic valve. Some other applications of computer simulations may involve computer simulations of the influence of the electromagnetic waves generated by the cell phone antenna over the human head.

All of the above computer simulations are usually performed using so-called finite element method. The unique feature of the finite element method is the existence of strict mathematical theory controlling the accuracy of the simulations carried out by using this method. Unfortunately, computer simulations performed by using the finite element method are expensive, and for difficult problems, it is still hard to obtain accurate solutions, even by using large parallel computers. This is because they require solutions of very large systems of linear equations, having several millions of unknowns, which requires the usage of expensive algorithms. Additionally, all of the above computer simulations suffer from significant numerical errors. A simplified explanation of the reason for such unfortunate situation is the fact that mathematical theory concerning the stability of these methods works in abstract mathematical spaces while computer simulations are performed with some approximations. In other words, the mathematical theorems formulated in the abstract spaces may not work anymore when transformed to computer finite dimensional binary world. The practical consequence of this fact is the low reliability of some computer simulations, which may result in catastrophic consequences, from unpredicted consequences of car crashes, through unexpected airplane failures, catastrophic civil engineering accidents, unexpected blood flow in arterial bypass, wrong behavior of the aortic valves, up to unreliable simulations of the influence of the electromagnetic waves over the human head.

Prof. Leszek Demkowicz, a mathematician with Polish origins, working at The University of Texas at Austin invented in 2010 new method called DPG (from Discontinuous Petrov-Galerkin). The DPG method can perform in a fully automatic mode computer simulations which are always stable and deliver solutions with the prescribed accuracy.

However, the DPG method is very difficult to implement, deep mathematical knowledge, often not accessible to engineers working on simulations, is required there. Additionally, the systems of linear equations generated by the DPG method are larger than those from classical finite elements and they are very expensive to compute on computers.

The goal of this research project is the development of a new computational paradigm called iGRM (isogeometric residual minimization method) mixing together the stabilization properties of the DPG method (resulting in stable computer simulations) extremely fast modern alternating directions solvers (allowing for performing large efficient computer simulations of difficult problems even on laptops), and modern isogeometric finite element method (allowing for smooth simulations of time dependent problems, integrated with Computer Aided Design systems). The iGRM may be of great interest to the computational community, since it allows for simple, fast, and smooth numerical simulations of difficult time dependent problems.