Abstract

The main aim of the project is the study of properties of certain algebraic objects (groups) through their analytical realisations (representations). Algebraic objects are often associated with discrete world which is rigid, whereas functions and operators are understood as flexible in some sense. Kazhdan's property (T) forms a bridge between these worlds: a group has property (T) if all operators representing the elements of the group satisfy certain (analytical) rigidity condition. A group with property (T) can be (equivalently) characterised by the fixed point condition: every isometric action of the group on a Hilbert space has a fixed point.

Although original definition used the language of irreducible unitary representations, one can often prove property (T) without the knowledge of unitary dual. E.g. property (T) can be deduced form the spectrum of the (smooth) Laplace operator on a manifold whith fundamental group satisfying the property. Moreover, the property is related to specturm of (algebraic – group-wise) Laplace operator in the group algebra, which facilitates purly algebraic approach: the existence of positive solution to a certain system of equations (and inequalities) in the group algebra is equivalent to property (T). Due to this connection the analytical form of rigidity can be reduced to algebraic computations.

In this project we want to approach property (T) for finitely presented, discrete groups from the computational point of view. The algebraisation of the condition allows a formulation in terms of existence of a solution o certain system. A search for such solution can be performed by mathematical programming (*semi-definite optimisation*). A solution to such optimisation problem approximates the true (exact) solution of the system of equations defining property (T). The search for the solution can be facilitated by a computer: indeed, there exist a few specialised softwares (*solvers*) with the main purpose of finding the solution of the appropriately coded optimisation problem. Additionally, the computational method as described allows the extraction of approximate *Kazhdan constant* which constitutes a quantitative version of property (T). Since determining the constant by theoretical means is a challenging task, very little is known about the actual value of the constant, even for very simple groups. The proposed computational method is the only method known to provide a decent lower bounds on the Kazhdan constants for relatively simple groups. In some cases ($SL(n, \mathbb{Z})$ for $n \leqslant 6$), by using the method we were able to determine precisely the *order of magnitute* of Kazhdan constants.

Using the computational methods we will investigate another rigidity-related property, known as Shalom's property H_T, which generalises (in some cases) property (T).

Property (T) and more importantly Kazhdan constants have numerous application, including computational group theory (product replacement algorithm), construction of expander graphs, geometric group theory and many more.