

## **Description for the general public**

We focus our attention on some important physical phenomena, which we will indicate below and we will describe then the scientific language. Our job it is to prove formalized theorems. We can explain their meaning by referring to phenomena we describe. We will present them one by one.

We are interested in crystal growth. This topic is very important because of its applications to industrial processes. The semiconductor industry depends a constant supply of artificially grown crystals. However, our work does not immediately influence optimization of the industrial processes. It is interesting to notice that the same mathematical framework is used in constructing the image analysis algorithms for edge detection.

In the case we consider, the growth may occur only due to the interaction of the crystal with the container and this is what we are interested in. In particular, in the interior of the container there are no mass sources.

We noticed that the following simple and obvious statement has profound consequences: “the boundary of the crystal surface, touching the container moves with the velocity that depends on the entire surface”. It turns out that this sentence implies existence of a boundary layer near the wall of the container which moves ... regardless of the bulk! In particular, the boundary layer may lag behind the surface. This has enormous implications on the facet motion. This is exactly our goal: describe the influence of the container on the boundary layer and its interactions with the crystal facets.

Facets are of special interest to us, because they determine the shape of the surface and their motion is different from the behavior of the curved parts. This is why we are interested in facets. We want to know their interactions, but above all we wish to know what are the conditions guaranteeing that they will be not ‘damaged’ during evolution, i.e. they will not bent nor break.

Our interests are not restricted to the crystal growth. The other groups of problems we want to pursue we can describe as follows. It turns out that the subsurface transport of contaminants or river delta motion has a mathematical description, which looks (formally) like the ice melting problem. That is the set of governing equations looks the same, but the content is different. For the ice melting problem the differential operators are used, while for the contaminants/sediments transport the integro-differential operators are employed. They behave as if we computed a fraction of derivative. These are the so-called fractional Caputo derivatives.

The ice melting problem is well-studied, we know all the necessary tools for its analysis. We hope that their proper adjustment will do the job in the case of mentioned above transport problems. Our task is the development of those missing theoretical tools.

It is possible that  $\alpha$ , the order of the Caputo fractional derivative is known but up a certain accuracy. We can ask the question, how could we handle such a situation? The answer is: take into account all possible  $\alpha$  with the weights corresponding to their frequencies. As a matter of fact we face a completely new kind of problems, which calls for creation of new mathematical tools for their study.

In addition, we mention that we want to treat the subsurface transport differently than we will deal with the delta advancement problem. We want to create different toolboxes, so that we could look at these phenomena from different angles.

Finally, we would like to mention that these transport phenomena are important from the practical point of view. For instance, we would like to know how river sediments are moved at the potential site of a river dam. Similarly, we think it is good to know how the contaminants will be spread under surface in case of leak. We strongly believe that our research will bring a significant breakthrough in the process of solving these problems.