In how many ways one can arrange a given number of square boxes in a gutter in such a way that they do not slide down? In how many ways one can fill a given area using square puzzles, each having two convex and two concave knobs? These are sample questions from **combinatorics**, a field of mathematics which deals also with sudoku and other kinds of puzzles and riddles.

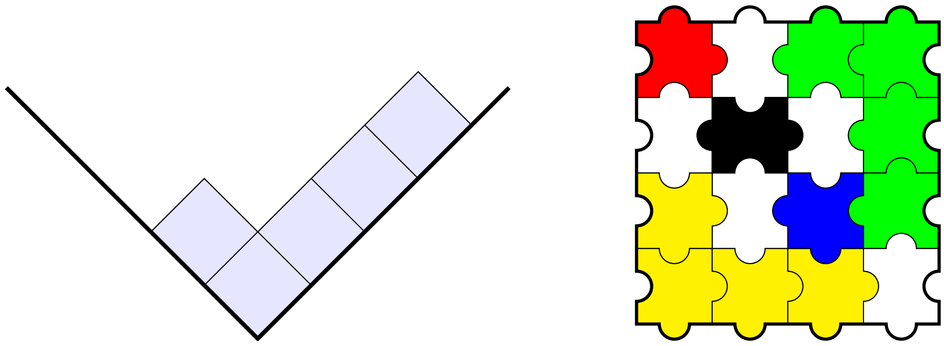

FIGURE 1. Sample combinatorial objects which are in the scope of the research project. The colors on the right figure correspond to the six types of the tiles.

Such questions become clearly more difficult as the number of boxes increases. At the same time, as the number of boxes tends to infinity, one can ask new, even more exciting problems: *if among all possible configurations of the boxes or puzzles we pick one by random, what can we say about the typical configuration sampled in this way?* If the square boxes from the first riddle are made of tiny quartz cristals, the above problem becomes a question about the typical shape of a large sandpile. Questions of this type are the subject of the **asymptotic combinatorics** and, at the same time, of this research project.

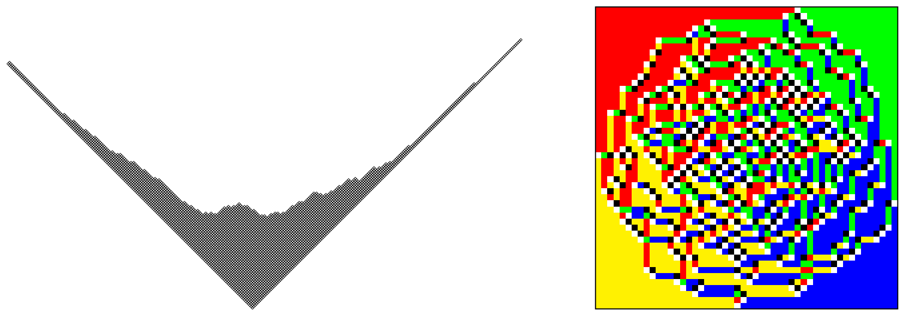

FIGURE 2. Very large counterparts of the combinatorial objects from Figure 1, sampled in a random way. The individual boxes are so small that they were not indicated on the figures.

The results of computer simulations (analogous to those from Figure 2) as well as of theoretical research indicate that in many such combinatorial models (as long as their size is big enough) a typical configuration concentrates around some limit shape with a probability which nears certainty. For exmaple, on the diagram on the right one can see some „frozen" monochromatic regions in the corners, while the circular area in the center looks like a chaotic „liquid". This phenomenon is very interesting from the viewpoint of mathematical physics and statistical mechanics, the subject of which is mathematically exact explanation and description of phase transitions (which we know well from everyday life as, for example, ice melting). One of the research goals of this proposal is to prove mathematically that this kind of phenomenon indeed occurs for a wider class of combinatorial models and to investigate the links of this phenomenon to other, seemingly distant disciplines of mathematics.

What happens if we add some more grains on the top of a sandpile? What is the shape of an avalanche resulting from removal of some sandgrains from the very bottom? These are examples of **dynamic problems**, which are in the special focus of this proposal. The answers for such questions of **dynamic asymptotic combinatorics** might be interesting not only for amateurs of sand crafting and sudoku lovers. The configurations of boxes in a gutter (called professionally *integer partitions*) appear naturally in surprisingly many various contexts. One of them is the *representation theory*, which investigates the ways in which abstract types of symmetries might take a concrete form. Since the Universe is full of various symmetries, this ubiquitous theory has multiple applications. In particular, if one day we manage to take full advantage of the computational power of *quantum computers* (or, conversely: if we manage to understand their limitations) this will happen most probably thanks to understanding such seemingly naive questions about the sandpiles.