

The aim of this project is a generalization of field theories that are used in theoretical physics to describe matter and fundamental interactions to the case of topologically compact phase spaces. In such a novel framework, the usual linear phase space of a given field theory is considered to be only an approximation, working for sufficiently small excitations of the field above its vacuum state. The compact phase space is assumed to be locally a linear space, which allows to recover the correct low energy limit.

The proposed approach will be applied to both bosonic and fermionic fields as well as gauge fields. This will allow to construct generalizations of the Standard Model, characterized by compact phase spaces. We expect that the compactness of phase space, by preventing a field from achieving configurations of an arbitrary high energy, will naturally regularize the theory in the ultraviolet limit. An especially interesting case on which we would like to concentrate our research is the spherical phase space, equivalent to the phase space of angular momentum or spin. In this situation there exists a possibility to treat the known field theories as low energy approximations of certain spin systems, especially the ones considered in condensed matter physics.

Such a possibility has been put forward by the authors of this project and investigated in the simple case of a scalar field. In particular, it has been shown that the continuous XXZ Heisenberg model, in the linear phase space limit and with the anisotropy parameter tending to zero, reduces to the relativistic Klein-Gordon field. Finding the similar spin system generalizations of the fundamental interaction theories will be one of our objectives.

The discussed field theoretical framework actually arose in the context of quest for the quantum theory of the gravitational interaction. A long term goal is a generalization of the gravitational field to the case of compact phase space. It is conceivable that in the similar way as introducing the curved geometry of spacetime changed our understanding of classical gravity, introducing the field phase space with nontrivial topology and geometry will allow to built a consistent quantum theory of gravitation.

We are convinced that the planned research will shed new light on many problematic issues associated with the Standard Model as well as quantum gravity, such as the UV divergences and renormalizability or the hierarchy problem. In particular, as the preliminary results for a scalar field have shown, the compactness of phase space has an impact on the vacuum energy density and in this way may be connected with the value of the cosmological constant. Moreover, it has also been demonstrated that there are promising perspectives for applying fields with compact phase spaces in cosmology. The considered framework, by providing a bridge between field theory and condensed matter physics, will undoubtedly generate a lot of new ideas and directions to explore. At the current stage, one can already point out a variety of potential applications, including investigation of systems with very large field amplitudes, simulations of field theories on quantum computers, utilization of field theoretical methods in condensed matter physics, construction of analog condensed matter models of gravity and extension of the phenomenology of quantum gravity, as well as applications in the interdisciplinary problems, for example neural field models. And, undoubtedly, much, much more, as realization of this project should demonstrate.