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## **Harmonic analysis related to differential, difference and differential-difference operators**

The classical harmonic analysis on  $\mathbb{R}^d$  is related to the Laplace operator

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \dots + \frac{\partial^2}{\partial x_d^2}.$$

With the operator  $\Delta$  there is an associated heat semigroup  $H_t f = h_t * f$ , where  $h_t(x)$  denotes the celebrated Gauss-Weierstrass kernel given by

$$h_t(x) = (4\pi t)^{-d/2} e^{-\frac{|x|^2}{4t}}.$$

Various objects in PDE's, harmonic and Fourier analysis are related to the operator  $\Delta$  and the semigroup  $H_t$ . Let us mention only: harmonic functions, the Fourier transform, Sobolev spaces, maximal operators, Hardy spaces, Hilbert/Riesz transforms, Riesz and Bessel potentials, radial Fourier multipliers, and BMO spaces.

Most of the objects mentioned above can be defined for operators different than  $\Delta$ . In this project we shall study some differential operators (e.g. Bessel, Laguerre, Grushin), differential-difference operators (Dunkl), and difference operators (discrete Laplacian). All of these operators generate linear semigroups of operators on spaces  $L^p(X, d, \mu)$ , where  $X$  denotes a space,  $d$  is a metric on  $X$ , and  $\mu$  - a non-negative measure.

One of the main objects of interest for us will be Hardy spaces  $H^p$  associated with the operators mentioned above and other. Our goal is to characterize these Hardy spaces by maximal functions, atomic decompositions, Riesz transforms, and by the system of conjugate harmonic functions in the case of the Dunkl operator. Moreover we shall study mapping properties of some operators (e.g. Hilbert/Riesz transforms and multiplier operators) on the spaces  $L^p$  and  $H^p$ .

Harmonic analysis related to the operators investigated in this project is studied by many mathematicians from all over the world. It seems that the number of publications and doctoral theses in this area of mathematics is growing in the last years. The theorems obtained in this project shall develop significantly the theory of function spaces, operators, and semigroups of linear operators. In this research, we plan to collaborate with other mathematicians, e.g. from Poland, Spain and France.