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Coupling of the Multiscale FEM with the Discontinuous Petrov-Galerkin approach

New materials are one of the most important factors influencing new technologies development. Such materials exhibit much better performance than traditional ones. A group of special attention are composites, i.e. materials made of at least two constituents and characterized globally by better parameters than any of the constituents separately. They are designed in such a way that the most desired properties of the constituents (e.g. high strength and small self-weight) were obtained. The key part of the design process are expensive and time consuming laboratory experiments. Nowadays, they are aided with numerical calculations that enable to shorten the computation time and lower the design cost as a result. Since the overall, macroscopic properties of materials depend on their microscopic features, it is extremely important to deeply understand, model and control influence of the micro-scale properties on the apparent macroscopic phenomena and processes in order be able to tailor designed composite response efficiently. However, such a modeling, called a multiscale one, is a challenging task, primarily due to the complex micro-structure of real materials and severe deformations at this level. Thus, the most advanced numerical methods and special multiscale techniques are required in order to obtain reliable results within a reasonable time.

The primary objective of the project is to improve efficiency and reliability of multiscale modeling and to contribute to better understanding relation between micro structure and macro properties of real materials. In our research project we tackle a problem, which is extra demanding due to the lack of the periodicity. Namely, we deal with materials with irregular micro-structure (arbitrary distribution of the constituents). Thus, the new numerical technique will help designers of new composites by offering a virtual (digital) material. The model will be also validated by laboratory measurements.

The main numerical method used nowadays in engineering is the finite element method (FEM). One assumes that the analyzed domain is divided into a set of its small subdomains, i.e. a mesh of finite elements. Each of such elements is characterized with respective material constants. One has to generate such a fine mesh that even the smallest inclusion was covered with a number of elements. In practice, even this step is very complex. Particularly in the case of 3D analyses. It is due to the fact that inclusions may have irregular shapes. Solution of the problem defined on such a fine mesh is usually infeasible or extremely time consuming.

The multiscale FEM (MsFEM), which is the approach we develop, requires significantly coarser mesh than described above. Thus, computations can be performed in the accepted time. Idea of the method is to assess effective characteristics of such a coarse mesh elements on the basis of the fine mesh element characteristics. Presented approach enables significant reduction of the computational time with a smart numerical treatment of the information on the complex micro-structure. Additional advantage of the MsFEM is that assessment of the effective coarse mesh element characteristics are independent. Namely, after coarse mesh generation we perform auxiliary computations within coarse elements separately. Thus, they can parallelized (simultaneous performance for each elements).

The finite element method uses two sets of functions - test and trial ones. Typically, these two sets are the same. In the case of the MsFEM, assessment of effective characteristics is, in fact, evaluation of the special functions that are modified versions of the standard ones used in computations. These modifications reflect the distribution of composite constituents.

The second aspect of the developed approach is the application of the discontinuous Petrov-Galerkin method (DPG). It uses different sets of test and trial functions. Indeed, for a given set of trial functions corresponding set of optimal test functions is assessed. It is optimal in a sense of convergence rate of results.