Summary of the project:
"Geometric and analytic properties of ordinary differential equations"
for the general public

In the project it is planned to study holomorphic foliations (phase portraits in the complex domain) and to work on two associated with them conjectures: the center problem for the Abel foliation defined by the equation $y' = P'(x)y^2 + Q'(x)y^3$ and foliations with Liouvillian first integral (which we want to express in terms of more elementary functions).

The next subject is the problem of limit cycles for real polynomial vector fields (the XVI-th Hilbert problem) in particular situations. These particular situation are following: a perturbation of a vector field with a Darboux first integral, and a perturbation of a field with a contour composed of separatrices of saddles.

We plan to study normal forms for singularities of vector fields. Recently we succeeded in introducing a new method. Now we want to show that the obtained forms are not analytic in general.

We will continue investigations of hypergeometric equations associated with so-called multiple zeta values. We want to compute a series of coefficients of a 6-th order equation for a generating function associated with $\zeta(3)$ using methods of oscillating integrals.

We will study coefficients of so-called 3-term relations for orthogonal polynomials and multiply orthogonal polynomials. We have shown that these coefficients obey spacial differential equations with respect to a parameter,. We plan to generalize previous relations with so-called Painlevé equations.

We will look for a chaotic dynamics in 3-dimensional Lotka–Volterra system (for populations of three competing species).

We want to study special situations in the dynamics of a rigid body around a fixed point. They are perturbations of completely integrable cases (like the Lagrange case), where we will study periodic trajectories, i.e., beyond the assumptions of the KAM theorem.

We also plan to study the N body problem.